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## Numerical method for determining the allowable medium temperature during transient operation of a thick-walled boiler element in power plant

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**Abstract** Secure and cost-effective power generation has become very important nowadays. Care must be taken while designing and operating modern steam power plants. There are regulations such as German boiler regulations (Technische Regeln für Dampfkessel 301) or European Standards that guide the user how to operate the steam power plants. However, those regulations are based on the quasi-steady state assumption and one dimensional temperature distribution in the entire element. This simplifications may not guarantee that the heating and cooling operations are conducted in the most efficient way. Thus, it was important to find an improved method that can allow to establish optimum parameters for heating and cooling operations. The optimum parameters should guarantee that the maximum total stresses in the construction element are in the allowable limits and the entire process is conducted in the shortest time. This paper summarizes mathematical descriptions how to optimize shut down process of power block devices. The optimization formulation is based on the assumption that the maximum total stresses in the whole construction element should be kept within allowable limits during cooling operation. Additionally, the operation should be processed in the shortest time possible.

**Keywords:** Heat transfer; Thermal stresses; Optimization; Power plant

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# 1 Accuracy evaluation of numerical method based on the analytical solution for simple thick-walled element

Let's consider a thick-walled pipe that has a uniform initial temperature  $T_0$ , the inner surface radius  $r_1$  and the outer surface radius  $r_2$  (Figs. 1, 2).

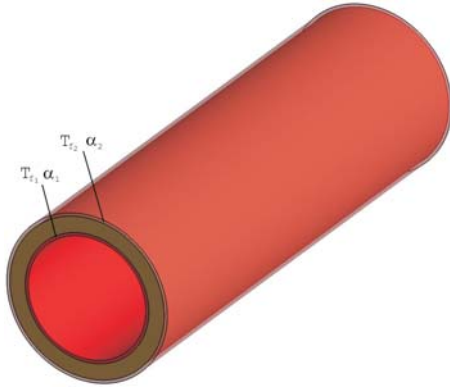


Figure 1. Geometry of the thick-walled pipe.

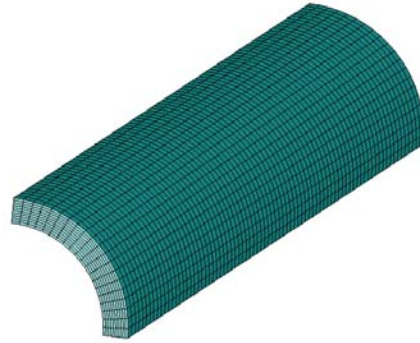


Figure 2. Geometry of the thick-walled pipe divided into 8-node finite elements.

Next, let's assume that the fluid enters the inner space of the pipe that has a constant temperature  $T_{f_1}$ . The internal pressure of the fluid  $p_1$  remains constant during the heating operation. The outer surface is exposed to ambient air at a temperature  $T_{f_2}$ . The heat on the inner and outer surface is exchanged with the constant convective heat transfer coefficient  $\alpha_1$  and  $\alpha_2$ , respectively. Under this condition, the temperature distribution  $T$  through the pipe wall in the radial direction can be obtained from a one-dimensional heat conduction equation. By assuming constant thermophysical properties, the temperature distribution is governed by the following equation [4,5]:

$$\left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) a = \frac{\partial T}{\partial t}. \quad (1)$$

The boundary conditions are as follows

$$\lambda \frac{\partial T}{\partial r} = \alpha_1 (T - T_{f_1}) \quad \text{for } r = r_1, \quad (2)$$

$$-\lambda \frac{\partial T}{\partial r} = \alpha_2 (T - T_{f_2}) \quad \text{for } r = r_2. \quad (3)$$

The initial condition is

$$T = T_0 \quad \text{for } t = 0. \quad (4)$$

Finally, substituting (2), (3) and (4) to (1) we obtain temperature distribution in the thick-walled pipe over time [5]:

$$\begin{aligned}
 T(r, t) = & T_{f_1} + (T_{f_2} - T_{f_1}) \left[ \frac{\ln \frac{r}{r_1} + \frac{\lambda}{\alpha_1 r_1}}{\ln \frac{r_2}{r_1} + \frac{\lambda}{\alpha_1 r_1} + \frac{\lambda}{\alpha_2 r_2}} \right] - \\
 & - \pi \sum_{n=1}^{\infty} \frac{(T_{f_1} \alpha_1 - T_{f_2} G_n \alpha_2) f(\mu_n, r)}{\alpha_1^2 + \lambda^2 \mu_n^2 - G_n^2 (\alpha_2^2 + \lambda^2 \mu_n^2)} e^{-a \mu_n^2 t} + \\
 & + \frac{\pi^2}{2} \sum_{n=1}^{\infty} \frac{\mu_n^2 f(\mu_n, r) e^{-a \mu_n^2 t}}{\alpha_1^2 + \lambda^2 \mu_n^2 - G_n^2 (\alpha_2^2 + \lambda^2 \mu_n^2)} \int_{r_1}^{r_2} -T_0 f(\mu_n, \eta) \eta d\eta, \quad (5)
 \end{aligned}$$

where  $\mu_n$  are the roots of equation [5]:

$$\begin{aligned}
 & [\alpha_1 J_0(\mu r_1) + \lambda \mu J_1(\mu r_1)] [\alpha_2 Y_0(\mu r_2) - \lambda \mu Y_1(\mu r_2)] - \\
 & - [\alpha_2 J_0(\mu r_2) - \lambda \mu J_1(\mu r_2)] [\alpha_1 Y_0(\mu r_1) + \lambda \mu Y_1(\mu r_1)] = 0, \quad (6)
 \end{aligned}$$

while  $f(\mu_n, r)$  and  $G_n$  are stated as follows

$$\begin{aligned}
 f(\mu_n, r) = & [\alpha_1 Y_0(\mu_n r_1) + \lambda \mu_n Y_1(\mu_n r_1)] J_0(\mu_n r) \\
 & - [\alpha_1 J_0(\mu_n r_1) + \lambda \mu_n J_1(\mu_n r_1)] Y_0(\mu_n r), \quad (7)
 \end{aligned}$$

$$G_n = \frac{\alpha_1 Y_0(\mu_n r_1) + \lambda \mu_n Y_1(\mu_n r_1)}{\alpha_2 Y_0(\mu_n r_2) - \lambda \mu_n Y_1(\mu_n r_2)}. \quad (8)$$

The total circumferential (thermal and mechanical) stress distribution is described by equation [5]:

$$\begin{aligned}
 \sigma_\varphi(r) = & \frac{\beta E}{1 - \nu} \left[ \frac{1}{r^2} \int_{r_1}^r T(r) r dr + \frac{r^2 + r_1^2}{r^2 (r_2^2 - r_1^2)} \int_{r_1}^{r_2} T(r) r dr - T(r) \right] + \\
 & + \frac{p_1 r_1^2}{r_2^2 - r_1^2} \left( 1 + \frac{r_2^2}{r^2} \right), \quad (9)
 \end{aligned}$$

where  $\beta$  is the thermal expansion coefficient,  $E$  is the Young's modulus and  $\nu$  denotes Poisson's ratio.

Total stress distribution in radial direction describes equation [5]:

$$\sigma_r = \frac{\beta E}{1 - \nu} \left[ -\frac{1}{r^2} \int_{r_1}^r T(r, t) r dr + \frac{r^2 - r_1^2}{r^2 (r_2^2 - r_1^2)} \int_{r_1}^{r_2} T(r, t) r dr \right] + \frac{p_1 r_1^2}{r_2^2 - r_1^2} \left( 1 - \frac{r_2^2}{r^2} \right). \quad (10)$$

Finally, the total stress distribution in longitudinal direction for not constrained pipe ends is described by equation [5]:

$$\sigma_l = \frac{\beta E}{1 - \nu} \left[ \frac{2}{r_2^2 - r_1^2} \int_{r_1}^{r_2} T(r, t) r dr - T(r, t) \right] + \frac{p_1 r_1^2}{r_2^2 - r_1^2}. \quad (11)$$

The temperature and stress history for the presented pipe was calculated using analytical and numerical techniques. In the performed analysis all material properties were assumed to be constant: Young modulus  $E = 196113$  MPa, thermal expansion coefficient  $\beta = 1.12 \cdot 10^{-5}$  1/K, specific heat  $C_p = 640$  J/(kg K), thermal conductivity  $\lambda = 29.7$  W/(m K), density  $\rho = 7750$  kg/m<sup>3</sup>, Poisson's ratio  $\nu = 0.3$ . The presented results were obtained for the following data: inner surface radius  $r_1 = 200$  mm, outer surface radius  $r_2 = 250$  mm, internal pressure  $p_1 = 8.43$  MPa, constant fluid temperature inside the pipe  $T_{f_1} = 300$  °C, constant ambient temperature  $T_{f_2} = 20$  °C, heat transfer coefficient on the inner and outer surface  $\alpha_1 = 2000$  W/(m<sup>2</sup>K) and  $\alpha_2 = 10$  W/(m<sup>2</sup>K), respectively.

Temperature and total circumferential stress histories on the inner and the outer surface, recorded during heating operation are shown in Figs. 3, and 4. At time point  $t = 1000$  s thermal stresses  $\sigma_\varphi$  on the inner and the outer surface are equal to -11.4 MPa and 7.5 MPa, respectively. The stresses coming from pressure on the inner and the outer surface are: 38.4 MPa and 30 MPa, respectively. Thus, as the system approaches the steady state, thermal stresses decrease and stresses coming from pressure play dominant role. It can be seen from Figs. 3 and 4 that numerical solution agrees very well with analytical solution.

## 2 Mathematical formulation of the proposed method

During the shut down process high stresses in the construction elements are created [8,9]. Thus, it is important to find optimum parameters that can maintain maximum total stress close to the stress limit  $\sigma_a$ , whereas

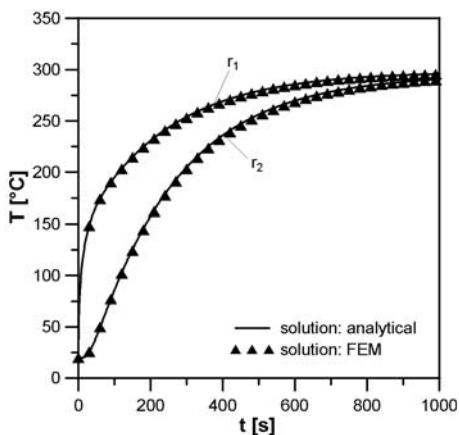


Figure 3. Temperature history on the inner and outer surface during heating operation.

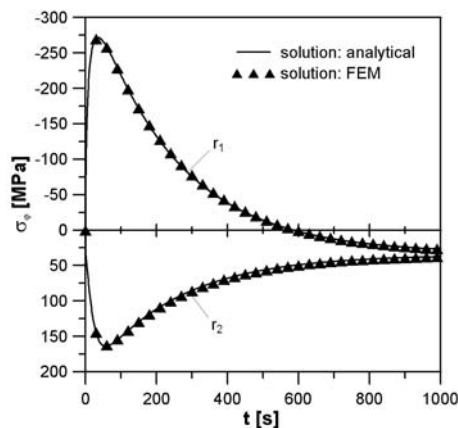


Figure 4. Total circumferential stress history on the inner and outer surface during heating operation.

the cooling time is shortest. These parameters can be found using the Levenberg-Marquardt algorithm [6,7]. Employing the proposed optimization procedure for outlet header shut down operation, two optimum parameters  $T_{f_1}$ ,  $v_{T_1}$  were found [10–12]. Parameter  $v_{T_1}$  denotes optimum rate of temperature change,  $T_{f_1}$  denotes optimum fluid temperature step. The searching of optimum parameters stopped when the following equation was satisfied:

$$\sigma_a - \sigma_{\max}(T_{f_1}, v_{T_1}, t_i) \cong 0, \quad i = 1, \dots, m, \quad (12)$$

where  $\sigma_{\max}$  denotes the highest value of total stresses in the whole construction element,  $m$  denotes the number of time points during the heating process. Again, minimizing the sum

$$S(x) = \sum_{i=1}^m [\sigma_a - \sigma_{\max}(T_{f_1}, v_{T_1}, t_i)]^2 \cong 0, \quad i = 1, \dots, m, \quad (13)$$

allows to establish the optimum parameters.

The parameters are updated at every  $k$ -th iteration step based on the following rule:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \delta^{(k)}. \quad (14)$$

The searched parameters of  $\mathbf{x}$  are denoted as  $x_1 = T_{f_1}$ ,  $x_2 = v_{T_1}$ , whereas

$$\delta^{(k)} = \left( \mathbf{H}^{(k)} + \lambda \text{diag} \mathbf{H}^{(k)} \right)^{-1} \left( \mathbf{J}^{(k)} \right)^T \left[ \sigma_a - \sigma_{\max} \left( x^{(k)} \right) \right], \quad k = 0, 1, \dots, \quad (15)$$

The  $\mathbf{H}$  and  $\mathbf{J}$  denote Hessian and Jacobian matrices, respectively. They are expressed as follows

$$\mathbf{J}^{(k)} = \frac{\partial \sigma_{\max}(x)}{\partial x^T} \Big|_{x=x^{(k)}} = \begin{bmatrix} \frac{\partial \sigma_1}{\partial x_1} & \cdots & \frac{\partial \sigma_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \frac{\partial \sigma_m}{\partial x_1} & \cdots & \frac{\partial \sigma_m}{\partial x_n} \end{bmatrix} \Big|_{x=x^{(k)}} \quad (16)$$

and

$$\mathbf{H}^{(k)} = \frac{\partial^2 \sigma_{\max}(x)}{(\partial x^T)^2} \Big|_{x=x^{(k)}} = \begin{bmatrix} \frac{\partial^2 \sigma_1}{\partial x_1 \partial x_1} & \cdots & \frac{\partial^2 \sigma_1}{\partial x_1 \partial x_n} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \frac{\partial^2 \sigma_m}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 \sigma_m}{\partial x_n \partial x_n} \end{bmatrix} \Big|_{x=x^{(k)}}. \quad (17)$$

The optimum parameters are found if the assumed convergence criterion

$$x_i^{(k+1)} - x_i^{(k)} \leq \varepsilon, \quad i = 1, \dots, n, \quad (18)$$

is fulfilled.

### 3 Optimization of the cooling process of the outlet header

In this section a numerical analysis of shut down operation of outlet header is presented. The geometry of the analyzed outlet header is shown in Figs. 5 and 6. The aim of the analysis was to find the optimum temperature history for the cooling operation. The optimum temperature history should assure that the maximum total stress in the outlet header remains within permitted limits and the process is not extended in time. The maximum permitted stress  $\sigma_a$  was evaluated based on the German boiler regulatios and equals 337 MPa. Using the proposed method two parameters defining optimum temperature history were found. These are:  $T_{f_1}$  and  $v_{T_1}$ , where  $T_{f_1}$  denotes step change of temperature at the beginning of operation,  $v_{T_1}$

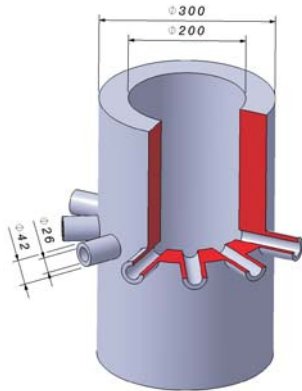


Figure 5. Geometry of the outlet header in mm.

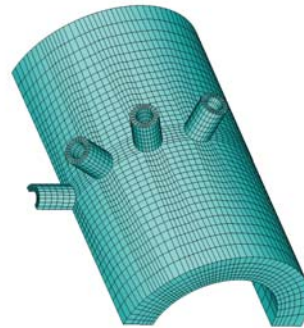


Figure 6. Geometry of the outlet header divided into 8-node elements.

denotes temperature change rate. Figure 7 shows the optimum temperature and pressure history, whereas pressure changes as a function of temperature. Next, simulation of cooling operation for presented in Fig. 7 temperature and pressure history was conducted. During this operation stresses in the outlet header were recorded, what is shown in Fig. 8. Maximum stress occurred on the inner surface, near the opening edges and reached 339.6 MPa at time  $t = 200$  s.

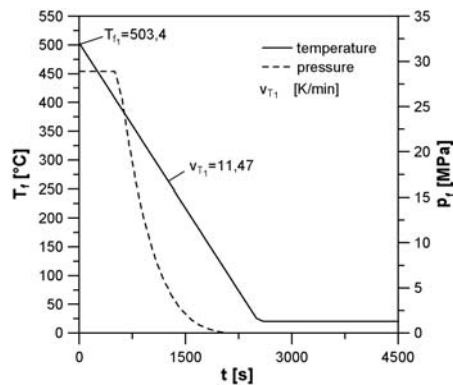


Figure 7. Temperature and pressure history based on the two optimum parameters.

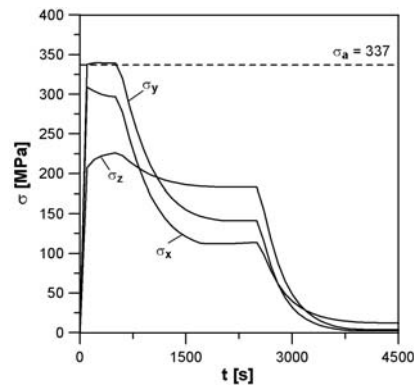


Figure 8. Stress history during cooling operation based on two optimum parameters.

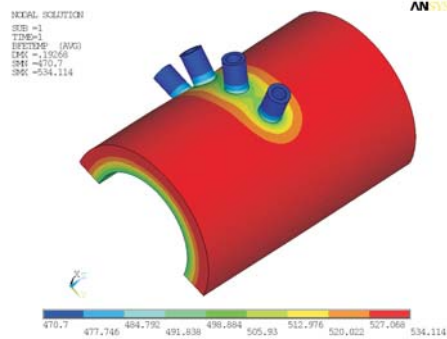


Figure 9. Temperature distribution in  $^{\circ}\text{C}$  at time  $t = 200$  s.

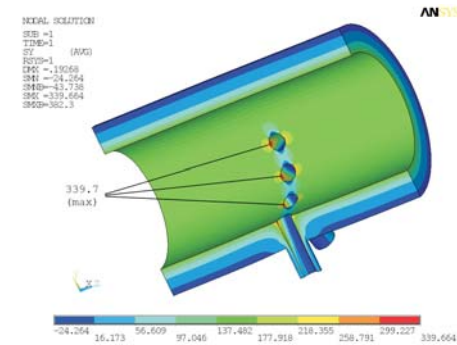


Figure 10. Total stress  $\sigma_y$  distribution in MPa for  $t = 200$  s.

Numerical finite element analysis proved that cooling operation of outlet header conducted with optimum parameters keeps the maximum total stress close to permitted stress. This can guarantee safe operation of power unit devices and reduces transient operation time.

## 4 Conclusion

In this paper a new optimization method based on the Levenberg-Marquardt algorithm was presented. This method allows to find the set of optimum cooling parameters so that the maximum total stresses in the construction element are within allowable limits during the cooling process. The accuracy of the numerical method was evaluated based on the analytical solution for the thick-walled component of simple shape. Next, numerical simulation of cooling operation of the outlet header was performed, where the total stresses during this process were monitored. It was shown that maximum total stresses during the entire transient operation were within the allowable limits. This approach allows to operate power unit devices in a safe way and reduces the shut down operation of power unit devices.

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