Spectral and integral methods of determining parameters in selected electric arc models with a forced sinusoid current circuit

ANTONI SAWICKI, MACIEJ HALTOF

Czestochowa University of Technology, Faculty of Electric Engineering
Armii Krajowej 17, 42-200 Częstochowa
e-mail: sawicki.a7@gmail.com, maciej@haltof.pl

Abstract: The paper discusses problems arising in attempts to accurately represent dynamic processes of an electric arc by means of simple mathematical models. It describes the properties of the universal Pentegov model, employing any shape of static voltage-current characteristics of an arc. Next, it presents spectral and integral measuring methods for determining arc parameters in the Mayr, Cassie and Pentegov models of the electric arc with a forced sinusoid current circuit, with the raising static characteristics of hyperbolic-flat and hyperbolic-linear shape. The influence is discussed of the random power supply disturbances on errors of determining the mathematical model parameters.

Key words: electric arc, Mayr model, Cassie model, Pentegov model, spectral methods, integral methods

1. Introduction

In the process of designing and controlling electrotechnological devices it is often necessary to deploy mathematical models of the electric arc. Such a model should be simple and at the same time it should represent with sufficient accuracy the dynamic voltage-current characteristics of the arc under the conditions of various kinds of disturbances and for a wide range of forcing current amplitude variation.

The popular Mayr and Cassie arc models [1, 2] are applicable to limited current ranges, which in turn depend on various external factors of the electric discharge [3, 4]. To overcome the limitations, attempts are being made to create extended mathematical models (e.g. Schwarz-Avdonin, Cassie-Mason, Reider-Urbanek, Kukiekov, Novikov) [5] or to combine simple arc models, for example by joining them in series (the Habedank model [6]) or in parallel (the hybrid model [7]). Extended arc models, however, are not free from drawbacks

related to determining a large number of parameters and non-linear static characteristics. In the hybrid model there are problems with determining the optimal forms of the weight function and of the damping factor function. In ad-hoc created complex mathematical models there are major difficulties with interpreting physical processes occurring in the arc. The boundary cases of such models can yield results significantly diverging from the actual physical phenomena. On the other hand, models of the Novikov-Shellhase type [8], deploying the static characteristics, represent the physical processes only very roughly, since they do not satisfy the energy balance equation. Because of this, it appears to be justified to utilise the universal Pentegov model (developed with Sydorets) [9] to represent the physical properties of arcs with any static characteristics.

2. Selected issues of representing arc working conditions in circuits of electrotechnological devices

Since the arc has to burn in a stable way despite its nonlinear static and dynamic characteristics, the power supply has to be appropriately selected. The arc characteristics are usually falling and because of that a real voltage power supply with even more steeply falling external characteristics is typically used. In electrotechnological devices of small power, such as welding devices, it is possible to use electronic power supply with almost vertical characteristics in the required voltage intervals. Determining parameters of arcs supplied from such practically ideal sources with sinusoid waveforms is the subject of this paper.

To solve the diagnostic and control problems, it is necessary to have online access to the parameters of arc mathematical models, which means that these parameters have to be monitored during the technological process. Despite the fact that in the arc there are numerous processes of electromagnetic, thermal, gas-dynamic, acoustic, optical and other types, it is very difficult to select suitable measuring sensors and to place them in the system due to several reasons, including very high temperature, relatively small size, difficult access to the plasma area, which is partly isolated, and strong non-homogeneity of plasma parameters. Sensors are subject to very intensive thermal wear and their life is short. Because of their low durability and unsatisfactory accuracy, indirect measurements are often preferred over direct measurements of non-electric quantities [10, 11]. Signals available for indirect measurements include current and voltage variation in time and sometimes also electrode separation. But even these quantities are sometimes computed rather than measured. All that means that a very small number of measurements must be sufficient for determining the static or dynamic arc characteristics, for creating mathematical models and determining their parameters as well as for assessing the operating conditions of devices.

The analytic, graphical and numerical methods known so far [12, 13] do not offer the possibility of determining the arc parameters on-line, since they do not deploy standard measurement equipment or applications emulating such equipment. In this study it is assumed that when an electric arc is supplied from a sinusoid current source, the deformed voltage wave-

form carries enough information to determine all the parameters required in the selected simple mathematical models of the electric arc.

In a real electrotechnological device there can be disturbances within power supply systems, electric arcs and measuring systems. Those occurring in the power supply can be deterministic or random. They deform supply current, which can become non-sinusoid. Such disturbances can be reduced or even eliminated by undertaking certain measures, e.g. using filters, or fast control systems. It is much more difficult to manage disturbances within the arc without affecting the technological process at the same time. Here the most frequently attested type of disturbances are changes in the arc column length caused by such factors as gas flow, magnetic fields, electrode vibrations, drop transitions of electrode material, variation in the weld pool size, etc.

We shall restrict our considerations of the dynamic arc characteristics to the phenomena occurring in the plasma column. Additionally, voltage drops near the electrodes were obtained by means of a popular method [14]. The influence of voltage drops on arc modelling depends on a number of factors and can be negligible in the case of long arcs.

As the arc discharge develops, the temperature increases and the gas becomes ionised to some degree. Consequently, the arc voltage decreases because its value depends on the arc column length, pressure, the gas flux around the column and other factors. The spatial structure of the electric arc can be divided into three regions: the cathode region, the anode region and the plasma column region. In each of these, energy is dissipated by means of different processes, since it is caused by different voltage drops. For a variable length free-burning arc, the static characteristics can be represented by a formula due to Nottingham

$$U_{stat}(I,L) = a + bL + \frac{c + dL}{I^n} = a + U_{col}(I,L),$$ (1)

where: U_{stat} – voltage on the arc, V; I – current; L – arc length; *a, b, c, d, n* – approximation constants (*a* – sum of the voltage drops near the electrodes), U_{col} – static voltage at the arc column. The influence of other factors on the voltage can be approximated by means of a similar formula [3]. For the most operating conditions of electrotechnological devices, variation in such parameters as the arc length, the pressure and gas flux around the column takes place much slower than variation in current. Because of that, these parameters are often assumed to be constant and the approximation $U_{stat}(I)$ is sufficient.

3. Problems of determining parameters in simple mathematical models of the electric arc

There are several experimental methods of obtaining dynamic parameters of an AC arc. They can be divided into the following types:
 1) Based on the natural periodic variation of current and voltage;
 2) Introducing disturbances into the forced current using additional current sources;
 3) Introducing disturbances into the conditions of energy dissipation from the arc column.

Introducing special disturbances requires extending the measuring systems by adding some electronic or electromechanical units and affects the arc burning conditions. This can cause undesirable responses from control systems and can become a source of additional systematic errors. In order to avoid that, it is necessary to be able to obtain online time constants of arc mathematical models during the regular operation of devices, especially that for many high power arc devices it is only possible to carry out measurements and experimental research *in situ*. This, in turn, makes it difficult, or even impossible to obtain static voltage current arc characteristics. To overcome the difficulties and to meet the requirements of contemporary control engineering, new methods are created of obtaining time constants and the other parameters of arc mathematical models, including approximations of the static voltage current characteristics. The analytic solutions of selected arc mathematical models in a steady state and with a sinusoid forced current can be represented as a Fourier series. On this basis, it is possible to obtain the relationships among the parameters [15]. Generally, the methods can be divided into:

– spectral, making use of selected harmonics of the electric quantities waveforms;
– integral, utilising the mean and effective (RMS) values of the electric quantities waveforms.

The spectral methods require special measuring equipment or special software for computer measuring systems. Relatively sensitive to interference and nor very accurate, the spectral methods do not require the numbers of neighbouring harmonics to be specified. The values of the harmonic amplitudes decrease as frequency increases and become comparable to interference, it is therefore advisable to choose low numbers of the harmonics. This is not, however, an absolute rule, since under the conditions of intensive disturbance of random or deterministic frequencies, selecting appropriate low or high harmonics can affect the accuracy of the results obtained. The spectral methods in their pure form enable experimental determination of time constants only, whereas the other arc parameters have to be obtained by means of the integral operators. The latter do not employ the spectral distribution, which makes them simpler and more accurate. Besides, they are suitable for determining the parameters of a greater number of models.

The Mayr model [1] of the electric arc makes it possible to represent fairly accurately the dynamic states caused by weak currents in the thermal plasma. It is assumed that the static voltage current characteristic is strictly hyperbolic

$$U_{col} = \frac{P_M}{I},\tag{2}$$

where: U_{col} – constant static voltage on the arc column; I – constant forcing current; P_M – constant power of the Mayr model. It is however apparent that in a number of cases with various external actions [16], such an approximation is very rough. The same is largely true of dynamic states. The Mayr model satisfies the energy balance equation and is represented as

$$\theta_M \frac{dg}{dt} + g = \frac{i^2}{P_M},\tag{3}$$

where: g – electric conductance of the arc column, θ_M – time constant of the Mayr model.

After the sinusoid current has been forced $i = I_{rms}\sqrt{2}\cos(\omega t + \varphi/2)$ it is possible to obtain a periodic solution of Equation (3) and to express it as the voltage variation $u_{col} = i/g$. The analysis of the Fourier series expansion of the voltage u_{col} leads to the spectral method of determining the model parameters.

The time constant of the Mayr model can be obtained from [17]

$$\theta_M = \frac{1}{4\omega}\left(\frac{1}{\chi_M} - \chi_M\right),\tag{4}$$

in which

$$\frac{U_{2k+1}}{U_{2k-1}} = \chi_M = \text{const.}\qquad k = 1, 2, 3,...\tag{5}$$

where: $2k + 1$, $2k - 1$ – are the numbers of the neighbouring odd harmonics; U – amplitude of a corresponding voltage harmonic of the arc column. The Mayr power can also be represented by means of the spectrum parameter χ_M [15]

$$P_M = U_{rms}I_{rms}\sqrt{\frac{1 - \chi_M^2}{1 + \chi_M^2}},\tag{6}$$

where: U_{rms} – effective (RMS) value of the column voltage.

As has been mentioned, the parameters of the arc mathematical model can be determined by means of the integral method, without determining the voltage spectrum. In the Mayr model, the time constant is

$$\theta_M = \frac{1}{2\omega\sqrt{\left(\dfrac{U_{rms}I_{rms}}{P}\right)^4 - 1}}\tag{7}$$

and the Mayr power

$$P_M = P,\tag{8}$$

where P – mean value of the momentary power function (active power) of the arc column.

The Cassie model [2] of the electric arc enables a fairly accurate representation of the dynamic states caused by strong currents in the thermal plasma. It is assumed that the static voltage current characteristics is horizontal

$$U_{col} = U_C\,\text{sgn}(I),\tag{9}$$

where: U_{col} – static voltage on the arc column; U_C – constant voltage of the Cassie model. With many external actions affecting the arc [4], the approximation offered by this model is also very rough and the same can be said about the dynamic states. The Cassie model satisfies the energy balance equation and is represented as

$$\theta_C \frac{dg^2}{dt} + g^2 = \frac{i^2}{U_C^2}, \tag{10}$$

where: θ_C – time constant of the Cassie model.

After the sinusoid current has been forced $i = I_{rms}\sqrt{2}\cos(\omega t + \varphi/2)$, it is possible to obtain a periodic solution of Equation (10) and represent it as the voltage variation $u_{col} = i/g$. The analysis of the voltage square u_{col}^2 expansion into the Fourier series leads to the spectral method of determining the model parameters.

The time constant of the Cassie model can be obtained from [15]

$$\theta_C = \frac{1}{4\omega}\left(\frac{1}{\chi_C} - \chi_C\right), \tag{11}$$

in which

$$\frac{A_{2(k+1)}}{A_{2k}} = \chi_C(k) = \text{const.} \quad k = 1, 2, 3, ... \tag{12}$$

where: $2k$, $2(k+1)$ – numbers of the neighbouring even harmonics; A – amplitude of the corresponding harmonic of the voltage square u_{col}^2 of the arc column. The Cassie voltage can be also expressed by means of the spectrum parameter χ_C

$$U_C = \sqrt[4]{\frac{2(u_{col}^4)_{av}}{3 - \chi_C^2}}, \tag{13}$$

where $(u_{col}^4)_{av}$ – mean (average) value of the voltage fourth power of the arc column.

The Cassie model parameters can be also determined by means of the integral method without determining the voltage spectrum. In such a case the time constant is

$$\theta_C = \frac{\dfrac{(u_{col}^4)_{av}}{U_C^4} - 1}{2\omega\sqrt{3 - 2\dfrac{(u_{col}^4)_{av}}{U_C^4}}}. \tag{14}$$

Then, the Cassie voltage can be obtained from

$$U_C = U_{rms} = \sqrt{(u_{col}^2)_{av}}. \tag{15}$$

4. Problems of determining the parameters of the Pentegov arc model with selected static characteristics

In a real arc with inertness, a stepped current change $i(t)$ induces a voltage impulse and a quasi-exponential change in resistance. In this model, the real arc is replaced by a hypo-

thetical arc. As in the other arc models, (e.g. Mayr, Cassie), the basic assumption in the Pentegov model [9] is the energy balance equation. However, in contradistinction to the real arc, the arc in the Pentegov model is electrically inertialess. Due to that, the arc resistance is obtained not from the real current but from a imaginary (virtual) lagging current $i_\theta(t)$, changing with the time constant θ and representing the real current $i(t)$. For high frequencies $f \gg 1/\theta$ of the AC supply, the arc thermal condition is defined by the effective current value. In the steady state, the state current $i_\theta(t)$ should be identical to the real current $i(t)$. All the isoenergetic states are characterised by one variable – the arc state current $i_\theta(t)$. By means of this variable, it is likewise possible to determine the parameters and dynamic characteristics of the arc model (Table 1).

The relationship between the squared state current and the squared real arc current is represented as a differential linear first-order equation

$$\theta \frac{di_\theta^2}{dt} + i_\theta^2 = i^2. \tag{16}$$

The Pentegov model represents the nonlinear element of the circuit, with the properties of being:
1) electrically balanced;
2) with the linear and stationary thermal inertia of the first order;
3) electrically inertialess.

Table 1. Definitions of momentary physical quantities in a real arc column and in the Pentegov model (U_a – voltage on the arc; a – sum of voltage drops near the electrodes)

Physical quantity	Real arc	Pentegov model
Electric current	$i(t)$	$i(t)$; $i_\theta(t)$
Momentary voltage on the arc column, $u_{col}(t) = U_a(t) - a =$	$u(i)$	$u(i_\theta) = R_{col}(i_\theta)i = \dfrac{U_{col}(i_\theta)}{i_\theta} i$
Supplied electric power to arc column, $P_{el}(t) =$	$P_{el}(i) = u(i) \cdot i = \dfrac{i^2}{g}$	$P_{el}(i, i_\theta) = R_{col}(i_\theta) \cdot i^2 =$ $= \dfrac{U_{col}(i_\theta)}{i_\theta} i^2$
Dissipated thermal power from the arc column, $P_{dys}(t) = P_{el}(t) - \dfrac{dQ(t)}{dt} =$	$P_{dys}(i) = P_{el}(i) - \dfrac{dQ(i)}{di} \dfrac{di}{dt}$	$P_{dys}(i, i_\theta) = U_{col}(i_\theta) \cdot i_\theta$
Resistance of the arc column, $r(t) =$	$\dfrac{u}{i}$	$\dfrac{U_{col}(i_\theta)}{i_\theta}$
Conductance of the arc column, $g(t) =$	$\dfrac{i}{u}$	$\dfrac{i_\theta}{U_{col}(i_\theta)}$
Arc plasma enthalpy, $Q(t) =$	–	$Q(i_\theta) = 2\theta \displaystyle\int_0^{i_\theta} U_{col}(i_\theta)di_\theta$

The advantages of this model include the possibility of utilising any approximation of the static voltage current characteristics and a constant value of the damping factor (referred to as the time constant).

Assume that in an arc circuit with the static characteristics (1), the forced current with a variable pulsation ω is applied

$$i = I_m \cos\left(\omega t + \frac{\varphi}{2}\right). \tag{17}$$

Then, the state current is

$$i_\theta^2 = I_{rms}^2\left(1 + \cos\varphi\cos 2\omega t\right), \tag{18}$$

where $I_{rms} = I_m/\sqrt{2}$ – effective current value. On the basis of definitions given by Pentegov [18], it is possible to obtain the effective voltage value and the mean arc resistance (Tab. 2). The value of the shift angle φ is bound with the arc time constant

$$\tan\varphi = 2\omega\theta. \tag{19}$$

On the basis of the assumptions, it is possible to obtain the time constant of the Pentegov model

$$\theta = \frac{1}{2\omega}\sqrt{\frac{1}{2\left(\dfrac{(i_\theta^4)_{av}}{I_{rms}^4} - 1\right)} - 1}, \tag{20}$$

where: $(i_\theta^4)_{av}$ – mean value of the column current fourth power. As can be seen, the time constant of the arc model is independent of the shape of the static characteristics. Formula (20) is:
– universal, since it does not refer to any specific static arc characteristic;
– not applicable, since it is expressed by means of the state current – a quantity inaccessible to measurement.

Table 2. Definitions of the measuring quantities in a real arc column in the Pentegov model

Physical quantity	Real arc column	Pentegov model
Effective current square, $I_{rms}^2 =$	$\dfrac{1}{T}\displaystyle\int_0^T i^2 \mathrm{d}t$	$\dfrac{1}{T}\displaystyle\int_0^T i_\theta^2 \mathrm{d}t$
Effective voltage square, $U_{rms}^2 =$	$\dfrac{1}{T}\displaystyle\int_0^T u_{col}^2 \mathrm{d}t$	$\dfrac{1}{T}\displaystyle\int_0^T U_{col}^2(i_\theta) \mathrm{d}t$
Mean value of momentary power, $P =$	$\dfrac{1}{T}\displaystyle\int_0^T u_{col} i \mathrm{d}t$	$\dfrac{1}{T}\displaystyle\int_0^T U_{col}(i_\theta)\cdot i_\theta \mathrm{d}t$
Mean value of resistance, $R =$	$\dfrac{1}{T}\displaystyle\int_0^T \dfrac{u_{col}}{i} \mathrm{d}t$	$\dfrac{1}{T}\displaystyle\int_0^T \dfrac{U_{col}(i_\theta)}{i_\theta} \mathrm{d}t$

The Mayr and Cassie models are special cases of the Pentegov models [18]. We shall subsequently demonstrate that for specific selected static voltage current characteristics it is possible to determine not only the time constant but also the other parameters of the model by means of the integral method.

The Pentegov arc column model with a static characteristics of the Ayrton (Mayr--Cassie) type offers the possibility of representing with satisfactory accuracy the dynamic states caused by weak and strong currents in thermal plasma. The shape of the static voltage current characteristics is assumed to be hyperbolic-linear (horizontal – Fig. 1). It can be represented by the formula

$$U_{col}(I) = U_{CP} \; \mathrm{sgn}(I) + \frac{P_{MP}}{I}, \tag{21}$$

where: U_{CP}, P_{MP} – constant approximation coefficients.

Assuming that the power $P_{MP} \approx 0$ W, the static characteristics obtained will be flat, as in the Cassie model. The voltage U_{CP} can be estimated on the basis of the static characteristics in the range of very high current ($U_{CP} \approx U_{col}(I)$). If it is assumed that the voltage $U_{CP} = 0$ V, the static arc characteristics obtained will be hyperbolic, as in the Mayr model. The power P_{MP} can be then determined by means of the coordinates (I_{MP}, U_{MP}) of any point belonging to the static characteristic curve ($P_{MP} = U_{MP} \cdot I_{MP}$).
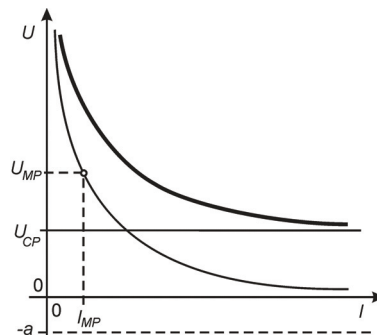

Fig. 1. A static voltage current characteristic with a hyperbolic-linear (horizontal) shape of the electric arc

To obtain the parameters of a sinusoid current-supplied arc experimentally, a method of three measurements is proposed, with three different effective values of the sinusoid current. On the basis of these measurements, the voltage and current effective values of the arc column as well as the arc column resistance mean values can be quantified: (U_{rms1}, I_{rms1}, R_1), (U_{rms2}, I_{rms2}, R_2), (U_{rms3}, I_{rms3}, R_3). Then, the sought parameters of the model are [15]:

$$P_{MP} = \frac{W_P}{W}, \tag{22}$$

$$U_{CP}^2 = \frac{W_U}{W}, \tag{23}$$

where the determinants W_P, W_C and W are defined as:

$$W = \begin{vmatrix} 2\left(R_1 I_{rms1}^2 - R_2 I_{rms2}^2\right), & I_{rms1}^2 - I_{rms2}^2 \\ 2\left(R_2 I_{rms2}^2 - R_3 I_{rms3}^2\right), & I_{rms2}^2 - I_{rms3}^2 \end{vmatrix}, \tag{24}$$

$$W_P = \begin{vmatrix} \left(U_{rms1} I_{rms1}\right)^2 - \left(U_{rms2} I_{rms2}\right)^2, & I_{rms1}^2 - I_{rms2}^2 \\ \left(U_{rms2} I_{rms2}\right)^2 - \left(U_{rms3} I_{rms3}\right)^2, & I_{rms2}^2 - I_{rms3}^2 \end{vmatrix}, \tag{25}$$

$$W_U = \begin{vmatrix} 2\left(R_1 I_{rms1}^2 - R_2 I_{rms2}^2\right), & \left(U_{rms1} I_{rms1}\right)^2 - \left(U_{rms2} I_{rms2}\right)^2 \\ 2\left(R_2 I_{rms2}^2 - R_3 I_{rms3}^2\right), & \left(U_{rms2} I_{rms2}\right)^2 - \left(U_{rms3} I_{rms3}\right)^2 \end{vmatrix}. \tag{26}$$

The time constant is

$$\theta_P = \frac{1}{2\omega\sqrt{\left[\frac{2R I_{rms}^2}{P_{MP}} + \left(\frac{U_{CP} I_{rms}}{P_{MP}}\right)^2 - \left(\frac{U_{rms} I_{rms}}{P_{MP}}\right)^2\right]^2 - 1}}, \tag{27}$$

where the associated values (I_{rms}, U_{rms}, R) come from one measurement selected out of the three.

The Pentegov arc column model with a hyperbolic linear static characteristics enables a fairly accurate representation of the dynamic states caused by weak and strong current in thermal plasma. It can be particularly useful for modelling an arc burning in selected gases, such as those containing argon, hydrogen or metal vapours, in gases under high pressure, with electrodes of a small diameter [4] etc. It is assumed that the static voltage current characteristics are of a hyperbolic-linear increasing shape (Fig. 2). They can be represented by the formula

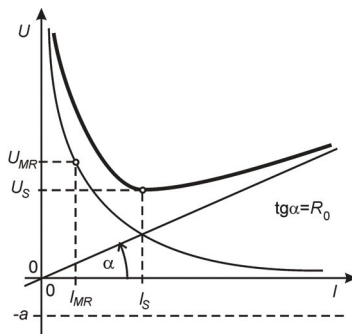$$U_{col}(I) = R_0 I + \frac{P_{MR}}{I}. \tag{28}$$

Fig. 2. A static voltage current arc characteristics of a hyperbolic-linear increasing shape

Assuming that the power $P_{MR} \approx 0$ W, the static characteristics will be linear, as of an element with a constant resistance R_0. The resistance R_0 can be estimated on the basis of the static characteristics in the range of very high current ($R_0 \approx U_{col}(I)/I$). Assuming that $R_0 = 0$ Ω, the static arc characteristics will be hyperbolic, as in the Mayr model. The power P_{MR} can then be determined from the coordinates (I_{MR}, U_{MR}) of any point belonging to the static characteristic curve ($P_{MR} = U_{MR} \cdot I_{MR}$).

If the location of the point (I_S, U_S) constituting the minimum of the static characteristics has been determined accurately, then its coordinates are

$$I_S = \sqrt{\frac{P_{MR}}{R_0}}, \tag{29}$$

$$U_S = 2\sqrt{P_{MR} R_0}. \tag{30}$$

On the basis of the coordinates it is possible to obtain the parameters of the static characteristics:

$$R_0 = \frac{1}{2}\frac{U_S}{I_S}, \tag{31}$$

$$P_{MR} = \frac{1}{2}U_S I_S. \tag{32}$$

The values R_0 and P_{MR} obtained experimentally in the way described above do not have to correspond to an optimal approximation of the static characteristics in a broad range of forcing current changes. Besides, this way of obtaining the parameters is possible on the condition that the static characteristic curve is known, which is not always the case. In this respect, the method offered below is much more promising.

We would like to propose an experimental method of obtaining the parameters of a sinusoid current supplied arc model. The method utilises the analytic solution of the Pentegov model and offers a possibility of determining its parameters by means of the integral method [15] and the quantities listed in Table 2:

$$P_{MR} = \frac{(U_{rms} I_{rms})^2 - P^2}{R I_{rms}^2 - P}, \tag{33}$$

$$R_0 = \frac{RP - U_{rms}^2}{R I_{rms}^2 - P}. \tag{34}$$

The time constant can be obtained from the formula

$$\theta_R = \frac{1}{2\omega\sqrt{\left[\dfrac{(R - R_0)I_{rms}^2}{P_{MR}}\right]^2 - 1}}. \tag{35}$$

5. Testing the effectiveness of various methods for determining
the parameters of selected mathematical models of the electric arc
with a forced sinusoid current and with disturbances

Industrial arc devices (electrothermal or welding devices) are typically characterised by high rated power. Such high loads are strongly nonlinear, with low values of the damping factor and random disturbances of the parameters, thereby generating strong interference in the supply grid. To protect the grid and other devices and to reduce the mutual detrimental effect of various electric devices, special filters, compensators, baluns, etc. can be used. Whether it is possible to connect an industrial device to the power grid depends on the ratio of the grid shorting power S_{zw} at the connection point to the equivalent power of the system of receivers. The following condition should be met

$$\sqrt{\sum_{i=1}^{n}\left(K_i^2 \cdot \sum_{j=1}^{m_i} S_{Tij}^2\right)} \leq S_{zw},$$ (36)

where: n – number of groups of devices of a given type; m_i – number of devices of a given type in a group, $K_i \gg 1$ – coefficient of each group of energy receivers. The division of devices into groups is based on: the arc type (AC or DC), presence or lack of reactive power compensation, pulsation of controlled rectifiers (6 or 12), etc. The frequencies of disturbances generated in the grid depend on the type of receivers, more specifically on their pulsation, nonlinearity, inertness, etc. Increasing the number of disturbance generating receivers causes an increase in the disturbance frequency in accordance with

$$f_L = f_1 \cdot 2n,$$ (37)

where: f_1 – frequency of oscillations generated by one device; n – number of electric devices.

The physically observable, measured and analysed higher harmonics of disturbances can be even of the 40th or 50th order with respect to the basic one. The condition (36) has to be strictly obeyed at the point of connecting the industrial plant to the power system in accordance with the relevant regulations and terms. The necessity to reduce disturbance within the power system of an industrial plant is motivated by the desire to reduce losses and to minimise the negative impact on the control devices. The electric arc can be a source of very strong disturbance and it can be itself affected by that originating from other sources.

In order to make the analyses of the selected models comparable, the considerations are limited to the influence of disturbance originating from power supply systems on the errors in determining the arc parameters. In our simplified considerations it is assumed that in power supply systems, there are random disturbances of different intensity and of constant frequency of the pseudorandom generator equal to 1000 Hz. The prescribed maximal value of the disturbance generator was equal to the percentage ξ_D of the supply current amplitude. In order to render a physically adequate representation of the disturbance, a cascade connection was created between the generator and the inertia unit of the first order with the time constant $T = 0.1$ ms.

We have used the MATLAB-Simulink software to do simulate the processes in a simple circuit containing a current supply source and a selected macromodel of the electric arc (Fig. 3). A similar series of experiments was carried out with the Mayr, Cassie and Pentegov models and described in [1, 2, 4, 16, 19]. In these studies, random disturbances of various intensity were introduced into the measuring system, the power supply or the arc column but only selected formulas were given and only some arc parameters were quantified without presenting a comparison of the methods applied.
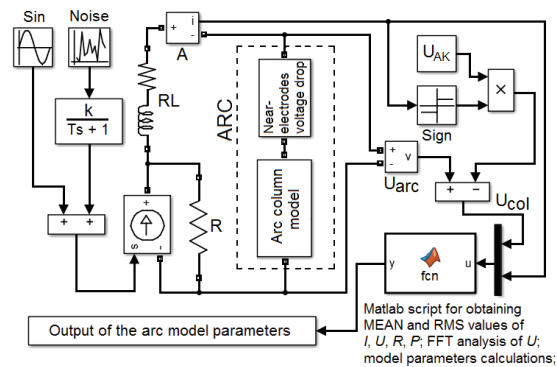

Fig. 3. Simplified schematic diagram of a system used for obtaining the parameters
of the electric arc mathematical models

Figure 4 presents the influence of the possible disturbance ξ_D of the supply current on the relative errors of determining the arc parameters (the power P_M and the time constant θ_M) in the Mayr model by means of the spectral method (δ_{PS}, $\delta_{\theta S}$) and the integral method (δ_{PI}, $\delta_{\theta I}$). It is evident that with an increase in ξ_D from 0 to 10%, all these errors increase almost linearly. The integral method yields the smallest error in obtaining the power and the greatest error in obtaining the time constant. Without disturbances in the current supply circuit, it is possible to determine the arc parameters with very small relative errors. In the spectral method, the ratio of the harmonics of the lowest order was applied (3/1).
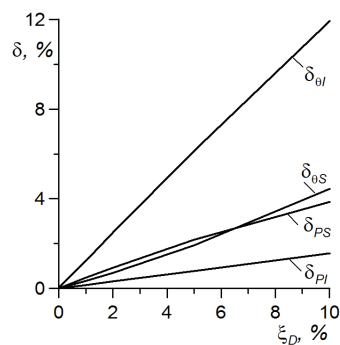

Fig. 4. Dependence of the relative errors in determining the Mayr model parameters
on the disturbance ξ_D of the supply current ($P_M = 60$ W, $\theta_M = 10^{-3}$ s, $I_{\max} = 5$ A, $f = 50$ Hz)

Figure 5a presents the influence of the disturbance ξ_D occurring in the supply current on the relative errors of determining the arc parameters (the voltage U_C and the time constant θ_C) in the Cassie model by means of the spectral method (δ_{US}, $\delta_{\theta S}$) and the integral method (δ_{UI}, $\delta_{\theta I}$). It can be seen that errors in the voltage obtained in the Cassie model by means of the two methods (spectral δ_{US} and integral δ_{UI}) are small and increase almost linearly with increase in the disturbance ξ_D. The errors in the time constant, on the other hand, depend on ξ_D very strongly and show opposite trends: the relative error of determining the time constant by means of the integral method ($\delta_{\theta I}$) increases and the one connected with the spectral method ($\delta_{\theta S}$) decreases, which can be explained by rounding errors in numerical calculations or by the spectral properties of the disturbance signal. If there are no disturbances in the current supply circuit, it is possible to determine the parameters with small relative errors, except for the time constant obtained by means of the spectral method. When the spectral method was used for analysing the voltage square of the arc column, the ratio of the amplitudes of the lowest-order harmonics was utilised (4/2). The experiment was subsequently repeated for the amplitude ratio of the higher harmonics (6/4). The curve $\delta_{\theta S}(\xi_D)$ was characterised by a steep drop, whereas the curve $\delta_{\theta I}(\xi_D)$ was represented by a steep rise up to 11%. These problems become less acute when the parameters are sought for an arc model with a greater time constant. Figure 5b presents the results of obtaining the Cassie model parameters by means of the spectral method for $\theta_C = 6 \cdot 10^{-4}$ s. When the amplitude ratio is taken into account for higher harmonics $\chi_{C4} = A_4/A_2$, then the voltage error δ_{U4} and the time constant error $\delta_{\theta 4}$ graphs are obtained, and if $\chi_{C6} = A_6/A_4$, then δ_{U6} and $\delta_{\theta 6}$ are obtained. As is evident, the error in determining the time constant depends on the ratio of the selected higher harmonics amplitudes.
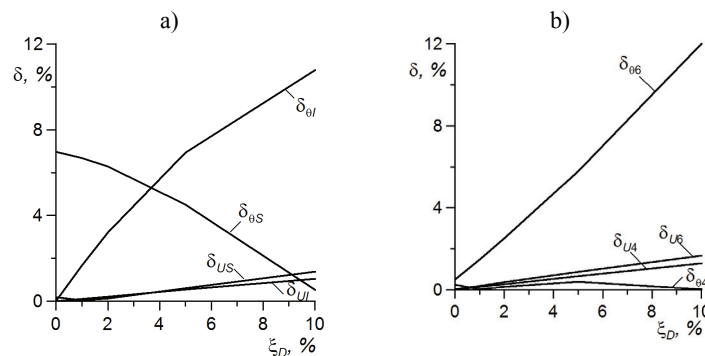

Fig. 5. Dependence of the relative errors in determining the Cassie model parameters on the disturbance ξ_D of the supply current ($U_C = 50$ V, $I_{max} = 150$ A, $f = 50$ Hz): a) $\theta_C = 3 \cdot 10^{-4}$ s, $\chi_C = A_4/A_2$; b) $\theta_C = 6 \cdot 10^{-4}$ s, $\chi_{C4} = A_4/A_2$, $\chi_{C6} = A_6/A_4$

Figure 6 presents the influence of the disturbance ξ_D introduced into the current supply source on the relative errors of determining the parameters in the Pentegov model (the power P_{MP}, the voltage U_{CP}, the time constant θ_P) with a hyperbolic-flat voltage current characteristic by means of the integral method (δ_{PM}, δ_{UC}, $\delta_{\theta P}$). As can be seen, all the errors increase almost linearly together with increase in the disturbance ξ_D. This method yields the smallest error for

the voltage U_{CP} and the greatest one for the time constant θ_P. If there are no disturbances in the current supply circuit, it is possible to determine the arc parameters with very small relative errors.
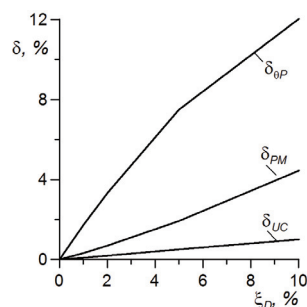

Fig. 6. Dependence of the relative errors in determining parameters of the Pentegov model with a hyperbolic-flat static voltage current characteristic on the disturbance ξ_D of the supply current ($U_C = 50$ V, $P_M = 60$ W, $\theta_P = 5 \cdot 10^{-4}$ s, $I_{max} = 200$ A, $f = 50$ Hz)

Figure 7 presents the influence of the disturbance ξ_D of the current supply source on the relative errors of determining the parameters in the Pentegov model (the resistance R_0, the power P_{MR}, the time constant θ_R) with a hyperbolic-linear increasing characteristic by means of the integral (δ_{R0}, δ_{PM}, $\delta_{\theta R}$). As can be seen, all the errors increase as ξ_D increases. This method yields the smallest error when it is applied for determining the resistance R_0. The errors are larger by five magnitudes for the time constant θ_R and the power P_{MR}. When there are no disturbances in the current supply circuit it is possible to quantify the arc parameters with very small errors.
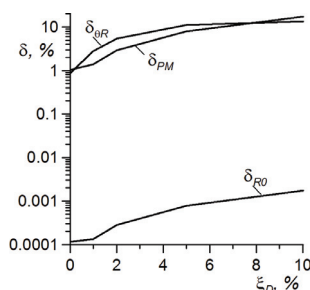

Fig. 7. Dependence of the relative errors in determining the parameters of the Pentegov model with the hyperbolic-linear increasing characteristics on the disturbance ξ_D of the supply current ($P_{MR} = 50$ W, $R_0 = 10$ Ω, $\theta_R = 3 \cdot 10^{-4}$ s, $I_{max} = 300$ A, $f = 50$ Hz)

It follows from the simulations that the integral methods produce smaller relative errors than the spectral methods, which means that the integral methods are more resistant to disturbance. The least accurate results were those obtained during determining very small values of the time constants, especially by means of the spectral method in the Cassie model.

In the simulations described above, the disturbances ξ_D introduced into the current supply source ranged from 0 up to 10%. In the ideal state of $\xi_D = 0\%$, the error in determining the parameters should be close to zero, since the methods applied are analytic solutions of mathematical models. If errors occur, they can result from inaccurate numerical calculations. In the circuits of real arc devices the level of disturbance is typically low. Disturbances of the arc column length represented as disturbances in the model parameters can be of greater signifi-

cance [19]. But even if they do occur, it is still possible to obtain the parameters of arc column models with errors not exceeding a few percent.

6. Conclusions

1) Applying the sinusoidal forced current in circuits with selected simple mathematical models of the electric arc makes it possible to develop effective measuring methods for obtaining parameters and static characteristics of these models.
2) As compared to the spectral methods, the integral methods are applicable to a greater number of mathematical models of the electric arc.
3) Errors in the parameters of the Mayr and Cassie models of the electric arc obtained by means of the integral methods are in the majority of cases smaller than those in the parameters obtained by means of the spectral methods.
4) Increase in the intensity of interferences in the power supply of an arc circuit contributed negatively to the accuracy of the methods of determining the arc model parameters. If the interferences do not exceed 10%, the relative errors of described methods for determining the mathematical arc models parameters are below 12%.
5) In the most cases it has been found that the increase of intensity of random noises in the power supply causes almost quasi-linear increase of the relative errors of the presented methods for obtaining the parameters of mathematical arc models.
6) If the arc is powered from a sinusoidal alternating current source, it is possible to obtain experimentally the static characteristics of selected mathematical models.

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