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Influence of fracture-matrix interaction on thermal front movement in fractured reservoir

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Abstract. Commercial exploitation of geothermal resources requires the disposal of large volumes of cooled brine in an environmentally acceptable way. Reinjection of cooled (i.e. spent) geothermal water has become a standard reservoir management strategy. Since injected fluids are typically much colder than the reservoir rock, this strategy results in the cooling of the region around the injection wells. Injected cool water may not have sufficient residence time in the reservoir to receive enough heat from surrounding hotter rock, resulting in temperature decrease at producing wells. Usually, the energy transport in geothermal reservoirs is calculated by use of sophisticated numerical simulators. In this paper we present analytical solution of simplified model of mass and heat transfer in fractured porous medium in one dimension, assuming constant rock temperature and neglecting small-scale effect connected with dispersion and heat conduction. The solution presented in this paper is applicable if thermal capacity of rock is high but the specific area is not sufficient for instant thermal equilibrium. This approach allows for better understanding the relation between fluid movement along the fractures and heat transfer between the rock matrix and fluid. Simple numerical experiments reported in the paper have shown the importance of specific surface area of naturally fractured rock, which influences the rate of exchange of heat between the fractures and the rock matrix.

Key words: geothermal reservoir, reinjection, thermal front movement.

Nomenclature

A – interfacial fracture-matrix specific area [m^2/m^3],
C – heat capacity [J/kg·K],
h – heat transfer coefficient [W/m^{2}K],
k – permeability [m^2],
K – thermal conductivity [W/m·K],
l – specific length [m],
Q – heat transfer between rock and fluid per unit of volume [W/m^3],
t – time [s],
T – temperature [°K],
u – Darcy velocity [m/s],
v – velocity [m/s],
x – distance [m],
s, w, z, L – geometrical dimensions [m].

Greek letters

ρ – density [kg/m^3],
ϕ – porosity [–],
α – coefficient defined by Eq. (10).

Subscripts

0 – initial,
in – injected,
r – rock,
f – fracture,
w – fluid (water),
T – thermal front.

1. Introduction

Commercial exploitation of geothermal resources requires the disposal of large volumes of cooled brine in an environmentally acceptable way. Due to environmental regulations this brine cannot be discarded at the surface and consequently must be injected underground, usually into the same geothermal reservoir. Besides environmental reasons, brine injection into the geothermal reservoir may provide additional benefits like maintaining reservoir pressure, enhanced heat recovery and reducing ground subsidence resulting from fluid withdrawal. On the other hand, since injected fluids are typically much colder than the reservoir rock, this strategy may result in cooling of the reservoir. Injected cool water may not have sufficient residence time in the reservoir to receive enough heat from surrounding hotter rock, resulting in temperature decrease at producing wells. This phenomenon is extremely important in highly fractured igneous rocks [1]. Estimating of heat recovery in naturally fractured geothermal reservoir needs a clear understanding of fluid movement along the fractures, conductive heat transfer in the rock matrix, convective heat transfer through fractures and heat transfer between matrix and fluid. Reinjection of cooled (i.e. spent) geothermal water has become a standard reservoir management strategy. In addition to meeting environmental requirements of fluid disposal, reinjection helps maintain reservoir pressure and increases energy extraction efficiency. Since injected fluids are typically much colder than the reservoir rock, this strategy

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will result in the cooling of the region around the injection wells. The cooled zone grows with time and eventually reaches production wells. After thermal breakthrough (i.e. arrival of the cold water front), the water temperature is no longer constant at the production wells, and may reduce the efficiency of the whole operation. It is thus important that we design a production–injection well system that will prevent cold water breakthrough before a specified time and it is essential that we determine the cold front velocity in the geothermal reservoir.

2. Previous work

Movement of the thermal front for a single-phase flow in homogeneous porous media was investigated by Bodvarsson [2]. Assuming, that thermal conduction is much smaller relative to convection, and neglecting heat exchange between pore system and rock matrix, he developed analytic solutions to the governing equations in the form of thermal shock front representing an abrupt change from the initial temperature to the injection temperature. The temperature front lags the fluid front by a constant depended on the ratio of rock/water volumetric heat capacities. An analytical model of thermal front propagation for one dimensional, linear flow with variable thermal properties of water-rock system was proposed by Stopa and Wojnarowski [3]. They concluded that thermal front moves slower than the injected water (as in the classical solution) and thermal front velocity differs from the front velocity obtained under the assumption of constant thermal properties by about 1–14% under a wide range of conditions. It was also shown that a temperature shock front does not form if hot water is injected into a colder reservoir.

The effect of thermal conduction was discussed by Shook [4], who concluded that neglecting conduction is generally a good assumption in non-fractured media. In fractured media, transverse heat conduction plays a more important role and mathematical model in this case should be improved. Most of known models assume local thermal equilibrium between matrix and fracture, which is not always valid. Another possibility is to model heat transfer between matrix and fracture through a source/sink term, this however requires additional assumptions of the model. In general, very limited number of analytical solutions exists in the literature for fractured porous media. The mass and heat transfer in fractured reservoirs are studied mainly numerically. One example of such approach is presented by Natarajan and Kumar [5]. Review of possible approaches existing in the literature is presented by Shaik et al. in the reference [6]. They concluded, that there exists a general lack of understanding the fluid flow through a natural fracture system and heat transfer between rock matrix and flowing fluid. Shaik et al. [6] investigates numerically the role of heat transfer between matrix and circulating fluid on energy production from fractured geothermal systems. In their approach matrix temperature and fracture temperature are treated individually. The energy balance equations for rock and fluid can be expressed as follows [6]:

$$\phi\rho_f C_f \frac{\partial T_f}{\partial t} + \nabla\cdot(\rho_f C_f u T_f) - K_f\nabla^2 T_f + Q = 0, \quad (1)$$

$$(1-\phi)\rho_r C_r \frac{\partial T_r}{\partial t} - K_r\nabla^2 T_r - Q = 0, \quad (2)$$

$$Q = hA(T_f - T_r). \quad (3)$$

Equation (3) is known as the Newton cooling law. Here, ϕ, ρ, C, T, u, K, Q, h, A represents the porosity, density, heat capacity, temperature, Darcy velocity of fluid, thermal conductivity, and heat transfer between rock and fluid, overall heat transfer coefficient and heat transfer area respectively. Subscripts f, r refer to fluid and rock respectively. Coefficient h represents heat transfer between fracture and matrix, and an interfacial fracture-matrix specific area. Both h and A are important parameters, influencing fracture-matrix interactions. While A can be estimated based on geometric relations between fractures and matrix blocks h is typically computed by harmonic averaging of matrix/fracture thermal conductivities [7], in the form of (4)

$$h = \frac{K_f K_r}{l_r K_f + l_f K_r}. \quad (4)$$

In numerical implementation l_f and l_m are taken as $w_f/2$ and $X_r/2$, with w_f denoting typical fracture aperture width and X_r matrix block length.

System of Eqs. (1) to (3) can be solved only numerically. Numerical results presented in the reference [6] shown that for time less than 10–15 years, the rock temperature slightly differs from its initial value under a wide range of conditions, resulting in higher than expected temperature of produced water. For instance, high values of heat transfer coefficients allow the fluid to capture more heat as it is circulated through the fractures, maintaining the high temperature of produced water. The same effect may be expected for slightly fractured reservoirs with large thermal capacity of the rock matrix and high thermal conductivity. Ahead of moving thermal front and also close to this front, temperature in matrix slightly differs from its initial value under a wide range of conditions. This justifies, that for some cases, close to the thermal front constant temperature in matrix may be assumed.

In this paper we present analytical solution of simplified model of mass and heat transfer in fractured porous medium in one dimension assuming constant rock temperature Tr, and neglecting small-scale effect connected with dispersion and heat conduction.

Mathematical formulation. Since the migration of fluid is faster along the high permeability fracture, transport of heat is assumed to be one dimensional along the fracture (Fig. 1) It is assumed, that in one dimension the principal transport mechanisms in the fracture is thermal convection. The coupling between the fracture and matrix is ensured by the continuity of the fluxes between them by assuming that the conductive flux from the fracture to the matrix may described by the Newton cooling law expressed by Eq. (3). Conduction and dispersion in the direction parallel to the fracture plane are assumed to be negligible.

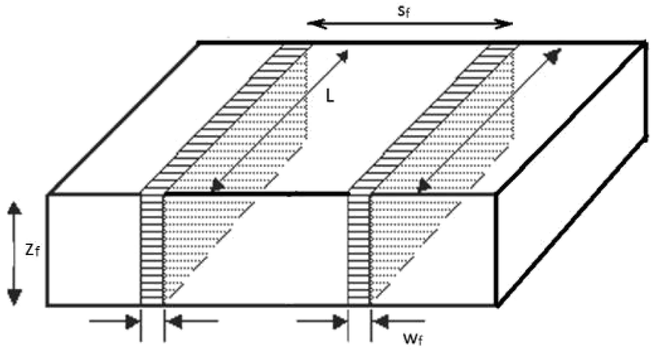

Fig. 1. Fracture dimension from modeling point of view

Assuming constant rock temperature and neglecting heat conduction, energy balance equation for fractures can be written as:

$$\frac{\partial}{\partial t}\left(\left((1-\phi)\,\varrho_r C_r+\phi\varrho_w C_w\right)T_f\right) + \frac{\partial}{\partial x}\left(\rho_w C_w u_w T_f\right)+h\cdot A\cdot\left(T_f-T_r\right)=0. \tag{5}$$

Equation (5) is a first-order and quasi-linear partial differential equation.

Boundary and initial conditions are:

$$T_f(x,t)=T_0 \quad \text{for} \quad x>0, \quad t=0, \tag{6}$$

$$T_f(x,t)=T_{in} \quad \text{for} \quad x=0, \quad t>0. \tag{7}$$

Equation (5) with conditions (6) and (7) can be solved using well known from mathematical literature method of characteristics. Assuming constant porosity, density and thermal properties, the characteristic family of Eq. (5) is described by the following system of equations:

$$\frac{dx}{dt}=\frac{u_w}{\varphi}\frac{\varphi\rho_w C_w}{(1-\varphi)\,\rho_r C_r+\varphi\rho_w C_w}=v_T. \tag{8}$$

And:

$$\frac{dT_f}{dt}=-\alpha\cdot(T_f-T_r). \tag{9}$$

Where:

$$\alpha=\frac{hA}{(1-\phi)\,\varrho_r C_r+\phi\varrho_w C_w}. \tag{10}$$

For $\alpha=0$, the characteristics are straight lines parallel to the plane $T=0$, whose slope dx/dt represents velocity of thermal front. For $\alpha=0$, the solutions of Eq. (5) generated by these straight lines therefore have the following fundamental property: if a solution at point x^* and time t^* has the value $T=T_o$, it will have the same T_o at point $x^*+\Theta\cdot v(T_o)$ and at time $t^*+\Theta$. In other words, the "slice" of porous medium in which the value of the temperature is T is displaced at a velocity $v_T(T)$. Consequently, the initial temperature profile is translated (i.e. moves away from the injection well) with a constant velocity v_T given by Eq. (8). This result is well known in the literature [4].

It follows that the temperature front lags behind the fluid front by a constant related to the ratio of rock/water volumetric heat capacities, and that there is an abrupt change from the initial temperature ahead of the front to the injected temperature behind it.

However, if $\alpha>0$, then from Eqs. (8) and (9) one can deduce that the temperature on the characteristic line is no longer constant and consequently the moving thermal profile will change its shape with time.

To solve the problem (5)–(7) one should observe that the characteristic curve passing through a point (0, 0, T_{in}) divides the characteristic curves family into two groups, which should be considered separately. The first group presents a family of characteristic curves starting from points (x^*, 0, T) for $T>0$, where x^* is a certain point on the x axis. The second group consists of curves starting from points (0, t^*, T_{in}) for a certain time $t^*>0$.

The equations for the characteristic curves will depend on these starting points and therefore x^* and t^* formally should be treated as variable parameters. The exact solution may be found in mathematical literature e.g. [8]. The wide discussion related to flow in artesian layer was also presented by Kaminski, Kordas and Siemek in the reference [9]. The formal solution of Eq. (5) by the method of characteristics is composed of two families of integral curves. The first is valid for $x>v_T t$ and the second for $x\le v_T t$. Finally, the exact solution of Eq. (5) with the conditions (6), (7) is composed of two functions:

$$T_f(x,t)=T_r+(T_0-T_r)\exp(-\alpha t) \quad \text{if} \quad x>v_T t, \tag{11}$$

$$T_f(x,t)=T_r+(T_{in}-T_r)\exp\left(-\frac{\alpha}{v_T}x\right) \quad \text{if} \quad x\le v_T t, \tag{12}$$

where α is expressed by Eq. (10) and v_T is expressed by Eq. (8).

Assuming equal initial temperature of the rock and the fluid $T_r=T_0$, Eqs. (11) and (12) can be simplified to:

$$T_f(x,t)=T_0 \quad \text{if} \quad x>v_T t, \tag{13}$$

$$T_f(x,t)=T_0+(T_{in}-T_0)\exp\left(-\frac{\alpha}{v_T}x\right) \quad \text{if} \quad x\le v_T t. \tag{14}$$

For $t>0$, position of thermal front can by defined as $x_T=v_T t$. It may be observed from Eqs. (4) and (12), that for $\alpha>0$ the temperature at thermal front depends on thermal properties of rock and water, and on the heat transfer area, which may be represented by specific surface of rock. Specific surface varies strongly depending on rock structure and consequently the coefficient α in Eq. (9) may differ by orders of magnitude for different rocks [10].

Influence of specific surface of naturally fractured rock. Reservoirs with natural fractures differ from those having intercrystalline intergranular porosity in that the double porosity system strongly influences the movement of fluids [11]. The double porosity can be the result of fractures, joints, and/or solution channels within the reservoirs. Carbonate reservoirs with a vugular solution porosity system, exhibit a wide range of permeability. The permeability distribution may be relatively uniform or quite irregular. The double porosity reservoir with a uniform permeability distribution is analyzed as follows. A specific area of fractures Spv is defined as the internal surface area per unit of pore volume [11], where the surface area for n fractures is $n(2w_f L + 2z_f L) = 2n(w_f + z_f)L$, and the pore volume is $n(w_f z_f L)$, assuming that the fracture provides all of the storage and permeability. The specific surface area per unit pore volume is then:

$$S_{VP} = 2\left(\frac{1}{z_f} + \frac{1}{w_f}\right). \tag{15}$$

Since $1/w_f \gg 1/z_f$, Eq. (15) reduces to:

$$S_{VP} = \frac{2}{w_f}. \tag{16}$$

By use of commonly known formula for fracture permeability k_f,

$$k_f = \frac{\varphi_f w_f^2}{12}. \tag{17}$$

Equation (16) became:

$$S_{VP} = \sqrt{\frac{\varphi_f}{3k_f}}. \tag{18}$$

The constant "3" in Eq. (18) is specific to the shape of the fracture. For real fractured reservoirs, equations (18) can be generalized for all fracture shapes by replacing constant "3" by coefficient $K_{Tf} = K_{sf}\tau$, where K_{sf} is known as the shape factor and τ is tortuosity [11]. The specific surface area of real sediments may exceed theoretical estimate by as much as three orders of magnitude because of surface roughness. On the other hand, preferential flow through permeable pathways in heterogeneous sediments could reduce the effective specific surface area by as much as 5 orders of magnitude in extreme examples. The effective surface area also could be reduced by organic or mineral coatings [10].

As the source term Q in Eq. (4) represents exchange of heat for the time dt, between matrix and fractures, per unit of volume of bulk rock, then

$$A = \varphi S_{VP}, \tag{19}$$

where ϕ is bulk porosity of naturally fractured reservoir which depends on fractures spacing and should not be confused with fracture porosity in Eq. (15). For uniformly distributed regular fractures with spacing s_f the bulk porosity can be assessed as:

$$\varphi = \frac{w_f z_f L}{s_f z_f L} = \frac{w_f}{s_f}. \tag{20}$$

Results and discussion. For illustration of the presented solution we use a simplified approach as follows. Combining (16), (19) and (20) yields $A = 2/w_f$. For numerical example we use the data presented in Table 1.

Table 1

Parameters		Value	
Initial reservoir temperature (K)		400	
Heat capacity of rock (J/kg K)		1170	
Density of rock (kg/m3)		2820	
Heat capacity of fluid (J/kg K)		4200	
Heat transfer coefficient (W/m2K)		50	
Density of fluid (kg/m3)		900	
Injected water temperature (K)		300	
Water viscosity, (cp)		0.3	
Darcy velocity (m/day)		2	
Reservoir porosity		0.01	
Spacing of fractures (m), α (1/day)	Case no.	S_f	α
	1.	0.1	26.1
	2.	1	2.61
	3.	10	0.26
	4.	50	0.052
	5.	100	0.026

Velocity of thermal front movement resulting from (8) is 2.28 m/day. For the time period of 100 days the position of the front is then at 228 m. This case refers to $\alpha = 0$. Figure 2 presents fluid temperature profiles for different fractures spacing from Table 1, and for $\alpha = 0$. In Fig. 3 the temperature vs. time is presented, at the moving point referred to actual position of thermal front.

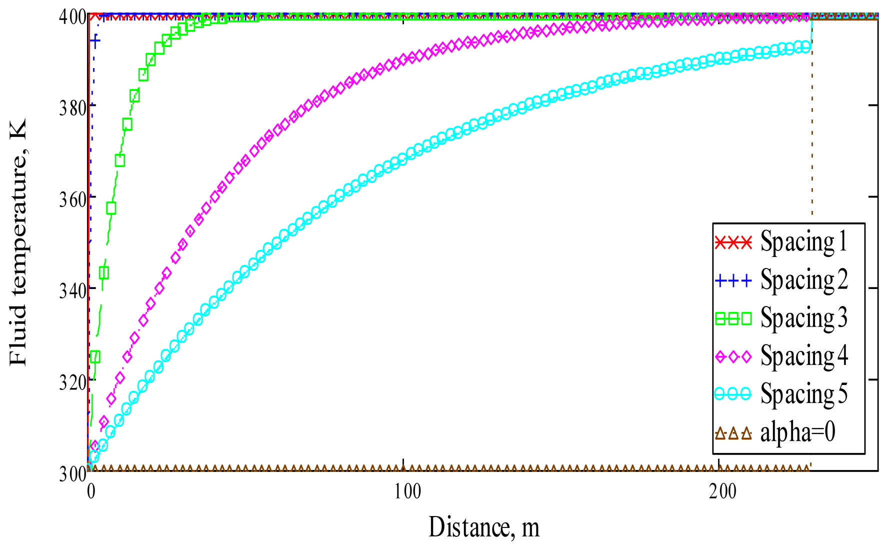

Fig. 2. Fluid temperature profiles for different fractures spacing from Table 1

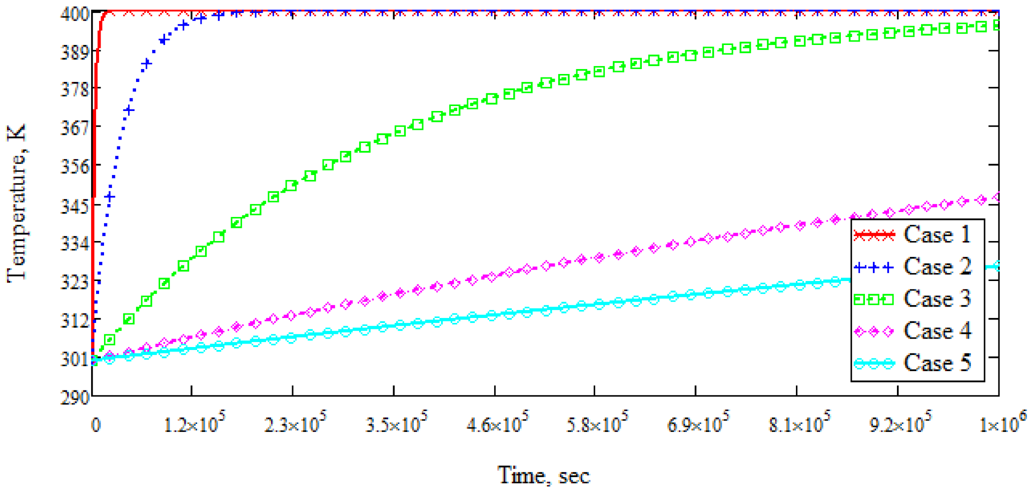

Fig. 3. Temperature vs. time at the moving point referred to actual position of thermal front

3. Conclusions

1. The specific surface area of naturally fractured rock influences heat transfer in two ways. Firstly, the specific area is related to fractures porosity and permeability. Secondly, the specific area also influences the source term in Eq. (1), which describes the rate of exchange of heat between the fractures and the rock matrix. This term implies that a larger specific area provides more access of the water to heat flow from the open fracture into the rock matrix.
2. Depending on geological properties of rock, two extreme cases may occur:
   - If heat exchange is intensive and heat capacity of rock is small, then rock receives the temperature of fluid flowing in the fractures. This phenomenon occurs if specific area of fractures is large (small fractures spacing) and the value of heat transfer coefficient is high. This case is relevant to the classical solution for thermal front movement.
   - If heat exchange is intensive and heat capacity of rock is large, then fluid flowing in the fractures receives the temperature of rock. This phenomenon occurs if specific area of fractures is large (small fractures spacing) and the value of heat transfer coefficient is high.

     For transient cases, if thermal capacity of rock is high but the specific area is not sufficient for instant thermal equilibrium, the solution presented in this paper is applicable. The heat exchange between rock matrix and the open fractures implies, that temperature at the thermal front is higher than the temperature of injected fluid.
   - The presented simplified solution assumes a constant temperature of the rock matrix. If this is not a case, only a numerical solution is possible.

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