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Adaptive output-feedback following control for time-delay systems

MARIÁN TÁRNÍK, JÁN MURGAŠ and EVA MIKLOVIČOVÁ

Adaptive control of the time-delay systems is presented in the paper. Despite the use of MRAC based design, only the model following (not perfect model following) is considered. The methods of a classical MRAC design are preserved to the maximum extent which allows further extensions of the algorithm such as the robust adaptive control modifications. The adaptive algorithm effectiveness is presented by means of illustrative examples.

Key words: adaptive control, model following, input time-delay

1. Introduction

The time-delay is ubiquitous to some extent in the control systems. It is caused, for example, by the processor computing time, sensor delay and obviously by the conveyor belt, etc. In general, there are many causes of the time-delay in the control systems.

Moreover, the high-order systems can be often modeled as a lower-order system with time-delay. There can be various reasons for such an approximation. In the papers [13, 5] the glucose dynamics is modeled using the second order system with the relative degree two and with the input time-delay. The second order system itself corresponds to the idealized glucose-insulin subsystem which represents the plasma glucose kinetics. The time-delay in this model is present due to the subcutaneous insulin administration and also due to the subcutaneous glucose concentration measurement.

Adaptive control is an effective control approach for uncertain or unknown systems and many applications can be found, e.g. [12, 1, 4, 3]. Numerous algorithms have been proposed for the state-delayed systems. This paper is focused on the systems with the input time-delay as discussed below. Moreover, the direct adaptive control algorithms, or the Model Reference Adaptive Control (MRAC) based algorithms, are considered in this paper.

The Authors are with Institute of Robotics and Cybernetics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Ilkovičova 3, 812 19 Bratislava, Slovak Republic. e-mail: {marian.tarnik, jan.murgas, eva.miklovicova}@stuba.sk

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The adaptive control algorithm presented in this paper falls within the class of algorithms which is characterized by the assumption that the controlled system time-delay is known. In this class of algorithms the two main approaches can be found. On the first approach the perfect model following is considered as a control design objective. In the other hand, the second approach considers only the adaptive model following (not the perfect model following) with a bounded error.

The first approach includes the adaptive controller proposed in [10, 2, 14]. It is a MRAC based algorithm for linear time-delayed SISO systems with relative degree $n^\star \leqslant 2$. In this algorithm the perfect model following in the sense of the classical MRAC is theoretically achieved. The controlled system time delay is assumed to be known and the Smith-predictor-like solution allows to adaptively predict the controlled system output. The predicted signal, among the other signals, is used as a feedback signal in the control law.

Mentioned algorithm components allow the perfect model matching in the classical sense. The controlled system output is predicted using a so-called distributed-delay block. Mathematically, it is a finite-time integral of the delayed controlled system input. In order to implement the control law, this integral has to be discretized. Such an approximation leads to many difficulties, see [11]. Due to the approximation only the model following can be practically achieved. Moreover, the obtained result has no theoretical background, since the approximation is not considered in the control algorithm design.

In [6] the adaptive following algorithm for plants with input and state delays is proposed. The algorithm utilizes the Smith predictor, which is based on the reference model transfer function rather than on the controlled system transfer function. Similarly as in the case of the classical Smith predictor use, this principle allows to pull the time-delay out of the control algorithm design procedure. As a consequence there is no need to use the distributed time delay block to establish a stability of the overall closed-loop system. The price for this advantage is that only the model following can be theoretically achieved. However, the obtained results have complete theoretical background and no uncommon approximations have to be considered for the algorithm implementation.

Nevertheless, the original authors consider mainly the state-feedback control laws [7], extends the basic idea for MIMO (multi input, multi output) systems [9], and propose the modifications for the robustness [8]. The control laws used in these algorithms are more or less modified in comparison with the classical MRAC based control law.

In this article we are interested in the output-feedback adaptive controller for SISO systems with input time-delay and with the relative degree $n^\star \geqslant 2$. The algorithms presented in the mentioned papers can not be directly extended for plants with higher relative degree due to the modified control law. Therefore the algorithm described in the next sections maintains the standard properties of the classical MRAC, mainly the structure of the control law, while the reference model based Smith predictor is used to cope with the time-delay.

2. Main Results

2.1. Preliminaries

Consider the controlled system in the form

$$y(s) = k_p \frac{Z_p(s)}{R_p(s)} e^{-\tau s} u(s) \tag{1}$$

where $y(s)$ and $u(s)$ are the output and input respectively. Further, $Z_p(s)$ is monic, Hurwitz polynomial of order m, $R_p(s)$ is monic and also Hurwitz polynomial of order n and k_p is so called high-frequency gain with known sign. It is assumed that the relative degree $n^{\star} = n - m \geqslant 2$ and the time-delay τ is known. The rest of the system parameters are unknown.

The classical MRAC based control law is considered in the form

$$u(t) = \Theta_1^{\mathsf{T}}(t)\nu_1(t) + \Theta_2^{\mathsf{T}}(t)\nu_2(t) + \Theta_3(t)y(t) + \Theta_4(t)r(t) \tag{2}$$

where $\Theta_c^{\mathsf{T}}(t) = \begin{bmatrix} \Theta_3(t) & \Theta_1^{\mathsf{T}}(t) & \Theta_2^{\mathsf{T}}(t) \end{bmatrix}$ and $\Theta_4(t)$ are the adapted parameters and the signals $\nu_1(t)$, $\nu_2(t)$ are the outputs of auxiliary filters introduced below. The reference signal $r(t)$ is the input to the reference model in the form

$$y_m(s) = W_m(s)r(s) = k_m \frac{Z_m(s)}{R_m(s)} r(s) \tag{3}$$

where $y_m(s)$ is the reference model output, k_m is the reference model high-frequency gain, $Z_m(s)$ is monic, Hurwitz polynomial of degree m_m, R_m is monic, Hurwitz polynomial of degree n_m, while the relative degree $n_m^{\star} = n_m - m_m = n^{\star}$. Further the reference model based Smith predictor is considered in the form

$$y_a(t) = [W_m(s)]\,\rho(t)\,(u(t) - u(t-\tau)) \tag{4}$$

where $y_a(t)$ is the output of the Smith predictor and $\rho(t)$ is the adapted parameter.

2.2. Quasi-controlled system

In order to formulate the control objective the quasi-controlled system is introduced. The output of the quasi-controlled system is the sum of the two signals: the controlled system output $y(t)$ and the Smith predictor output $y_a(t)$. The control objective is satisfied when the error

$$e_{a1}(t) = (y(t) + y_a(t)) - y_m(t) \tag{5}$$

is zero.

The controlled system (1) can be written in the form

$$\dot{x}(t) = Ax(t) + bu(t-\tau) \tag{6a}$$
$$\dot{\nu}_1(t) = \Lambda\nu_1(t) + qu(t-\tau) \tag{6b}$$
$$\dot{\nu}_2(t) = \Lambda\nu_2(t) + qc^{\mathsf{T}}x(t) \tag{6c}$$
$$y(t) = c^{\mathsf{T}}x(t) \tag{6d}$$

where (6b) and (6c) are the auxiliary filters with state vectors ν_1, $\nu_2 \in \mathbb{R}^{n-1}$, further $q \in \mathbb{R}^{n-1}$, $q^{\mathsf{T}} = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}$ and $\Lambda \in \mathbb{R}^{n-1\times n-1}$ is an arbitrary stable matrix. The matrices $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$ and $c \in \mathbb{R}^n$ are unknown.

Equations (6) can be written in the compact form

$$\dot{X}(t) = A_oX(t) + B_cu(t-\tau) \tag{7a}$$
$$y(t) = C_c^{\mathsf{T}}X(t) \tag{7b}$$

where $X^{\mathsf{T}}(t) = \begin{bmatrix} x^{\mathsf{T}}(t) & \nu_1^{\mathsf{T}}(t) & \nu_2^{\mathsf{T}}(t) \end{bmatrix}$ and matrices A_o, B_c and C_c have an appropriate form.

Similarly, the reference model can be represented in the form

$$\dot{X}_m(t) = A_cX_m(t) + \overline{B}_cr(t) \tag{8a}$$
$$y_m(t) = C_c^{\mathsf{T}}X_m(t) \tag{8b}$$

where $X_m(t)$ is the non-minimal state vector, and the matrices A_c, $\overline{B}_c$ and C_c are specified below. This implies that the Smith predictor (4) can be written in the form

$$\dot{X}_a(t) = A_cX_a(t) + \overline{B}_c\rho^{\star}\left(u(t) - u(t-\tau)\right) \tag{9a}$$
$$y_a(t) = C_c^{\mathsf{T}}X_a(t) \tag{9b}$$

Consequently the quasi-controlled system can be written in the form

$$\begin{aligned}\dot{X}(t) + \dot{X}_a(t) = A_oX(t) + B_cu(t-\tau)&\\ + A_cX_a(t) + \overline{B}_c\rho^{\star}\left(u(t) - u(t-\tau)\right)&\end{aligned} \tag{10a}$$
$$y(t) + y_a(t) = C_c^{\mathsf{T}}\left(X(t) + X_a(t)\right) \tag{10b}$$

The quasi-controlled system allows to formulate the standard MRAC control objective as follows. The control objective is to choose an adaptive law to adapt the parameters of the control law (2) so that all signals in the overall closed-loop system are bounded and the quasi-controlled system output $y(t) + y_a(t)$ tracks the reference model output $y_m(t)$. Particularly the perfect model matching is considered in the ideal case.

2.3. Ideal control law parameters

With regard to the equation (7) the control law (2) can be written in the form

$$u(t) = \Theta_c^{\mathsf{T}}(t)DX(t) + \Theta_4(t)r(t) \tag{11}$$

where the matrix D is in the form

$$D = \begin{bmatrix} c^{\mathsf{T}} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

In the ideal case the quasi-controlled system output equals the reference model output. In the view of equations (8) and (10), this implies that the matching conditions have the form

$$A_o + B_c \Theta_c^{\star\mathsf{T}} D = A_c \qquad B_c \Theta_4^{\star} = \overline{B}_c \quad \text{and} \quad \overline{B}_c \rho^{\star} = B_c \tag{12}$$

where the symbol * denotes the ideal parameters of the control law and Smith predictor.

2.4. Error equation

By adding and subtracting the terms

$$\pm \left(\overline{B}_c \rho^{\star} (u(t) - u(t-\tau))\right)$$

and

$$\pm \left(B_c \Theta_c^{\star\mathsf{T}} DX(t) + B_c \Theta_4^{\star} r(t)\right)$$

in the equation (10), further by defining the parameter errors $\widetilde{\Theta}_c(t) = \Theta_c(t) - \Theta_c^{\star}$, $\widetilde{\Theta}_4(t) - \Theta_4^{\star}$ and $\widetilde{\rho}(t) = \rho(t) - \rho^{\star}$, by considering the error equation (5) and the matching conditions (12), the dynamics of the error equation can be written in the form

$$\begin{aligned} \dot{e}_a(t) =& A_c e_a(t) + B_c \left(\widetilde{\Theta}_c^{\mathsf{T}}(t)DX(t) + \widetilde{\Theta}_4(t)r(t)\right) \\ &+ \overline{B}_c \widetilde{\rho}(t)\left(u(t) - u(t-\tau)\right) \end{aligned} \tag{13a}$$

$$e_{a1}(t) = C_c^{\mathsf{T}} e_a(t) \tag{13b}$$

To simplify the notation, $\widetilde{\Theta}^{\mathsf{T}}(t) = \left[\widetilde{\Theta}_c^{\mathsf{T}}(t) \quad \widetilde{\Theta}_4\right]$, $\omega^{\mathsf{T}}(t) = \left[(DX(t))^{\mathsf{T}} \quad r(t)\right]$ and $u_\tau(t) = u(t) - u(t-\tau)$ are introduced which implies that the control law can be written in the form $u(t) = \Theta^{\mathsf{T}}(t)\omega(t)$ and the equations (13) can be written in the form

$$\dot{e}_a(t) = A_c e_a(t) + \overline{B}_c \left(\frac{1}{\Theta_4^{\star}} \widetilde{\Theta}^{\mathsf{T}}(t)\omega(t) + \widetilde{\rho}(t) u_\tau(t)\right) \tag{14a}$$

$$e_{a1}(t) = C_c^{\mathsf{T}} e_a(t) \tag{14b}$$

Equations (14) relate the adapted parameter errors and the adaptation error $e_{a1}(t)$ through the transfer function $W_m(s)$. However $W_m(s)$ is not strictly positive real (SPR) transfer function since the relative degree $n_m^\star \geqslant 2$. In order to use a Lyapunov-based adaptive law design, the error dynamics has to be given by the SPR dynamical system. Therefore, well known augmented error method is used as follows.

For convenience the equation (14) can be rewritten by defining $\Theta_n^\mathsf{T}(t) = \begin{bmatrix}\frac{1}{\Theta_4^\star}\Theta^\mathsf{T}(t) & \rho(t)\end{bmatrix}$, $\Theta_n^{\star\mathsf{T}} = \begin{bmatrix}\Theta^{\star\mathsf{T}} & \rho^\star\end{bmatrix}$ and $\omega_n^\mathsf{T}(t) = \begin{bmatrix}\omega^\mathsf{T}(t) & u_\tau(t)\end{bmatrix}$, to the form

$$e_{a1}(t) = [W_m(s)]\left(u_n(t) - \Theta_n^{\star\mathsf{T}}\omega_n(t)\right) \tag{15}$$

where $u_n(t) = \Theta_n^\mathsf{T}(t)\omega_n(t)$. Further, the dynamical system (15) with transfer function $W_m(s)$ is assumed to have the relative degree $n_m^\star = 2$. However, the following procedure can be easily extended for higher relative degrees. Using the identity $(s+\rho)(s+\rho)^{-1} = 1$, where $\rho > 0$ is an arbitrary constant, the equation (15) can be written in the form

$$e_{a1}(t) = \left[W_m(s)(s+\rho)(s+\rho)^{-1}\right]\left(u_n(t) - \Theta_n^{\star\mathsf{T}}\omega_n(t)\right) \tag{16}$$

and consequently

$$e_{a1}(t) = [W_m(s)(s+\rho)]\left(u_f(t) - \Theta_n^{\star\mathsf{T}}\omega_{nf}(t)\right) \tag{17}$$

where $u_f(t) = \left[(s+\rho)^{-1}\right]u_n(t)$ and $\omega_{nf} = \left[(s+\rho)^{-1}\right]\omega_n(t)$.

Let the transfer function $W_m(s)(s+\rho)$ be chosen so that it is a strictly positive real transfer function. Further it is assumed that the signal $u_n(t)$ can be realized so that the signal $u_f(t)$ is in the form

$$u_f(t) = \Theta_n^\mathsf{T}(t)\omega_{nf}(t) \tag{18}$$

Substituting the equation (18) to the equation (17) leads to

$$e_{a1}(t) = [W_m(s)(s+\rho)]\left(\theta_n^\mathsf{T}(t)\omega_{nf}(t)\right) \tag{19}$$

where $\theta_n(t) = \Theta_n(t) - \Theta_n^\star$. The system (19) can be written in the state space form

$$\dot{e}_a(t) = A_c e_a(t) + \overline{B}_c\left[(s+\rho)\right]\theta_n^\mathsf{T}(t)\omega_{nf}(t) \tag{20a}$$

$$e_{a1}(t) = C_c^\mathsf{T} e_a(t) \tag{20b}$$

Since s is the Laplace operator the transformation $\overline{e}_a(t) = e_a(t) - \overline{B}_c\theta_n^\mathsf{T}(t)\omega_{nf}(t)$ can be introduced. Further, due to the fact that $C_c^\mathsf{T}\overline{B}_c = 0$, denoting $B_1 = A_c\overline{B}_c + \rho\overline{B}_c$, the system (20) can be written in the form

$$\dot{\overline{e}}_a(t) = A_c\overline{e}_a(t) + B_1\theta_n^\mathsf{T}(t)\omega_{nf}(t) \tag{21a}$$

$$e_{a1}(t) = C_c^\mathsf{T}\overline{e}_a(t) \tag{21b}$$

2.5. Adaptation law

In order to design the adaptive law for the parameters of control law (2) the adaptive law structure is assumed in the form

$$\dot{\theta}_n(t) = f(e_{a1}(t), \omega_{nf}(t)) \tag{22}$$

Since $W_m(s)(s+\rho) = C_c^\mathsf{T}(sI - A_c)^{-1}B_1$ is strictly positive real transfer function the Meyer-Kalman-Yakubovich lemma implies that there exists a matrix P, such that

$$A_c^\mathsf{T} P + PA_c = -Q \tag{23a}$$

$$PB_1 = C_c \tag{23b}$$

where the matrix $Q = Q^\mathsf{T} > 0$.

For the system consisting of equations (20a) and (22) the Lyapunov function candidate is considered in the form

$$V(t) = \bar{e}_a^\mathsf{T}(t)P\bar{e}_a(t) + \theta_n^\mathsf{T}(t)\Gamma^{-1}\theta_n(t) \tag{24}$$

where $\Gamma = \Gamma^\mathsf{T} > 0$ is an arbitrary matrix. The time-derivative of the function (24) can be after substituting (20a) written in the form

$$\begin{aligned}\dot{V}(t) =& \bar{e}_a^\mathsf{T}(t)\left(A_c^\mathsf{T} P + PA_c\right)\bar{e}_a(t) \\ &+ 2\left(\bar{e}_a^\mathsf{T}(t)PB_1\right)\theta_n^\mathsf{T}(t)\omega_{nf}(t) + 2\theta_n^\mathsf{T}(t)\Gamma^{-1}\dot{\theta}_n(t)\end{aligned} \tag{25}$$

The function (25) is negative semi-definite if $\dot{\theta}_n(t)$ is chosen in the form

$$\dot{\theta}_n(t) = -\Gamma\left(\bar{e}_a^\mathsf{T}(t)PB_1\right)\omega_{nf}(t) \tag{26}$$

Then it follows that

$$\dot{V}(t) \leqslant \bar{e}_a^\mathsf{T}(t)(-Q)\bar{e}_a(t) \tag{27}$$

Therefore the system (20a), (26) is neutrally stable relative to $\theta_n(t)$ and asymptotically stable relative to $\bar{e}_a(t)$.

The equations (23) imply that $\bar{e}_a^\mathsf{T}(t)PB_1 = \bar{e}_a^\mathsf{T}(t)C_c = C_c^\mathsf{T}\bar{e}_a(t)$ and further from (20b) it follows that $C_c^\mathsf{T}\bar{e}_a(t) = e_{a1}(t)$. Moreover $\dot{\theta}_n(t) = \dot{\Theta}_n(t)$ since $\Theta_n^\star$ is time-invariant. Therefore the adaptive law (26) can be written in the form

$$\dot{\Theta}_n(t) = -\Gamma e_{a1}(t)\omega_{nf}(t) \tag{28}$$

The adaptive law (28) ensures that the control law parameters are adapted so that the quasi-controlled system output $y(t) + y_a(t)$ tracks the reference model output $y_m(t)$. Simultaneously, all signals in the closed-loop system are bounded. From the previous result, i.e. (27) it directly follows that $\dot{\Theta}_n(t)$ is bounded. Therefore the signal vector $\omega_{nf}(t)$

is also bounded. Vector $\omega_{nf}(t)$ includes the signal $u_\tau(t)$ which is obviously also bounded, and it enters the stable Smith predictor (9). The signal $y_a(t)$ is therefore bounded. The adaptation error $e_{a1}(t)$, which is bounded, is given by the signals $y_a(t)$, $y_m(t)$ and $y(t)$. Therefore the controlled system output $y(t)$ is bounded. Since the controlled system is assumed to be stable, the control signal $u(t)$ is necessarily bounded. This concludes the discussion on boundedness of all closed-loop system signals.

From the practical point of view the adaptive law (28) consists of two components in the form

$$\dot{\Theta}(t) = -\mathrm{sign}\,(\Theta_4^\star)\,\Gamma_1 e_{a1}(t)\omega_f(t) \tag{29a}$$

$$\dot{\rho}(t) = -\gamma_2 e_{a1}(t) u_{\tau f}(t) \tag{29b}$$

where the matrix Γ has been assumed in the form

$$\Gamma = \begin{bmatrix} \Gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}$$

and in the view of the signals in (17), $\omega_f(t) = \left[(s+\rho)^{-1}\right]\omega(t)$ and $u_{\tau f}(t) = \left[(s+\rho)^{-1}\right] u_\tau(t)$ have been used.

2.6. Adaptation error signal

The assumption on the signal $u_n(t)$ in the section 2.4 is disadvantageous. This assumption implies the need for the signal of adapted parameters time derivative. For example the higher time derivatives are not directly available. Nevertheless, since the equation (19) is the standard error equation used in the classical direct adaptive control, the well-known augmented error method can be used in this case. Therefore it can be shown that the following equation can be used to obtain the adaptation error signal while the results of the previous sections still hold.

$$\begin{aligned} e_{a1}(t) = {} & [W_m(s)]\,\theta_n^\mathsf{T}(t)\omega_n(t) \\ & - [W_m(s)L(s)]\left(\left[L^{-1}(s)\right]\Theta_n^\mathsf{T}(t)\omega_n(t)\right. \\ & \left. -\Theta_n^\mathsf{T}(t)\left[L^{-1}(s)\right]\omega_n(t)\right) \end{aligned} \tag{30}$$

where in general the polynomial $L(s)$ is chosen so that the transfer function $W_m(s)L(s)$ is strictly positive real. In the particular case mentioned above $L(s) = (s+\rho)$.

The equation (30) allows that the signal $u_n(t)$ preserves its form, i.e. $u_n(t) = \Theta_n^\mathsf{T}\omega_n(t)$. Therefore, the standard control law (2) preserves its form and the adaptation error is augmented as given by the equation (30).

3. Illustrative example

The basic properties of the adaptive algorithm presented in the previous section are shown by means of the illustrative example. The overall scheme of the algorithm is shown in Fig. 3. As mentioned the control law is in the standard form $u(t) = \Theta^{\mathsf{T}}(t)\omega(t)$, where the signal vector $\omega(t)$ is in the form

$$\omega^{\mathsf{T}}(t) = \begin{bmatrix} \nu_1^{\mathsf{T}}(t) & \nu_2^{\mathsf{T}}(t) & y(t) & r(t) \end{bmatrix}$$

The Smith predictor, which can be seen as a part of the control law is also in the standard form (4) as discussed above, i.e.

$$y_a(t) = [W_m(s)]\,\rho(t)\,(u(t) - u(t-\tau))$$

The use of augmented error method implies the equation (30), which allows to generate the adaptation error for the adaptive laws as shown in the top of Fig. 3.

To illustrate the adaptive control algorithm, consider the controlled system in the form (1), where the high-frequency gain $k_p = 3$, the polynomial $Z_p(s) = 1$, $R_p(s) = s^2 + 4s + 2$ and the time-delay $\tau = 3,5$ [sec]. The reference model is given by the transfer function

$$y_m(s) = W_m(s)r(s) = \frac{1}{s^2 + 2s + 1}r(s)$$

where the reference signal $r(t) = \mathrm{sign}(\sin(2\pi f_r t))$, with $f_r = \frac{1}{25}$ [sec^{-1}].

The adaptation gains Γ_1 and γ_2 are the main parameters of the adaptive control algorithm which determine the transient adaptation process. However, the parameters such as the matrix Λ and the constant ρ have also indirect influence on the adaptation process. In this example the following parameter values have been chosen. Since $n - 1 = 1$ the parameter Λ is scalar and $\Lambda = -1$. Further $\rho = 1$ which ensures that the transfer function $W_m(s)(s+\rho)$ is strictly positive real.

Two cases of the adaptation gain choice are considered to illustrate the difference. In the first case the adaptation gains are chosen as

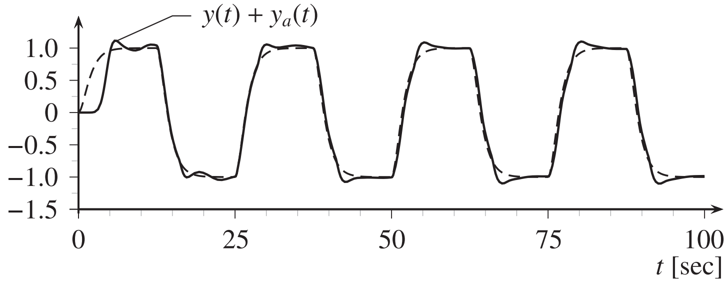
$$\Gamma_1 = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \quad \text{and} \quad \gamma_2 = 10$$

The results of the simulation experiment are shown in Fig. 1.

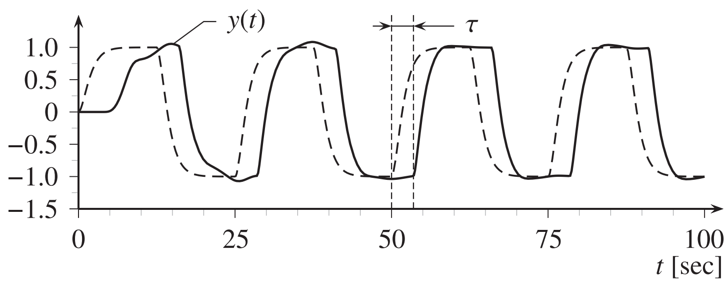
In the second case following adaptation gain values are used

$$\Gamma_1 = \begin{bmatrix} 0.3 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix} \quad \text{and} \quad \gamma_2 = 35$$

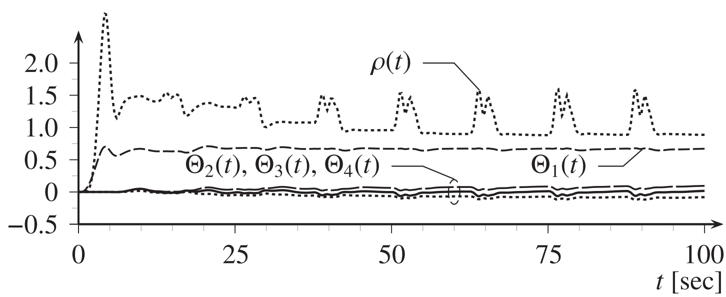
and the simulation experiment results are shown in Fig. 2.



(a) Quasi-controlled system output $y(t)+y_a(t)$ in comparison with the reference model output $y_m(t)$ (dashed line corresponds to the signal $y_m(t)$).



(b) Controlled system output $y(t)$ in comparison with the reference model output $y_m(t)$.

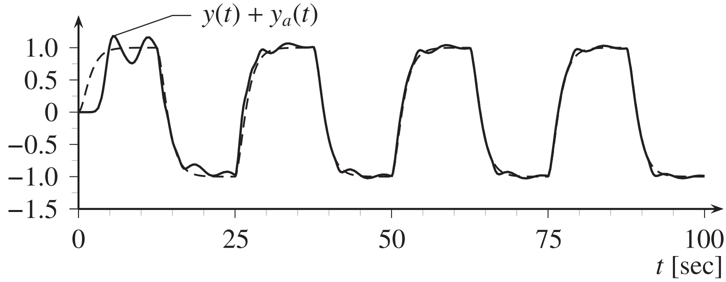


(c) The adapted parameters.

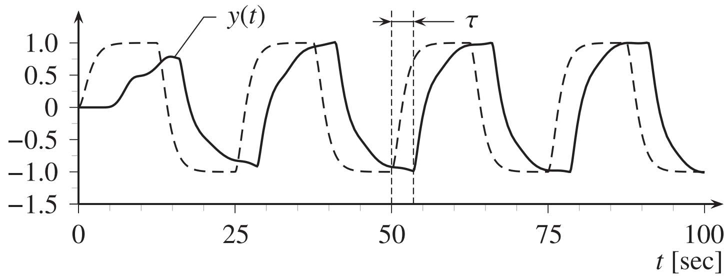
Figure 1: Results of the simulation experiment — case No. 1.

4. Conclusion

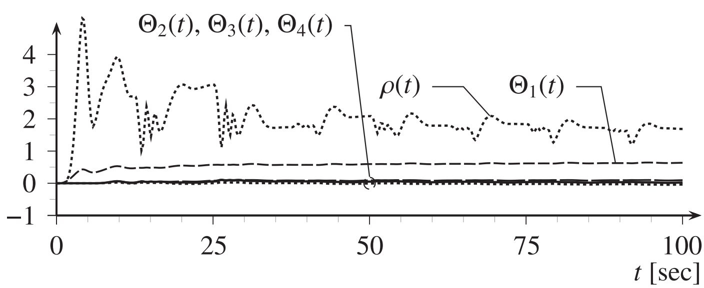
The presented algorithm ensures the model reference adaptive following with bounded error. For the quasi-controlled system the perfect model matching is achieved. However, from the practical point of view, the output of the actual controlled system



(a) Quasi-controlled system output $y(t)+y_a(t)$ in comparison with the reference model output $y_m(t)$ (dashed line corresponds to the signal $y_m(t)$).



(b) Controlled system output $y(t)$ in comparison with the reference model output $y_m(t)$.



(c) The adapted parameters.

Figure 2: Results of the simulation experiment — case No. 2.

$y(t)$ should satisfy certain quality requirements. Nevertheless, the control performance requirements even for the signal $y(t)$ are given by the reference model and by the reference signal. The choice of these two algorithm components determines the ideal behavior of the signal $y(t)$.

In the case of adaptive control, the transient adaptation process has to be also considered in the control performance evaluation. The choice of the adaptation gains determines the behavior of the signals in the transient process. As shown in the illustrative example in the section 3 the adaptation process can be tuned with the focus on the actual

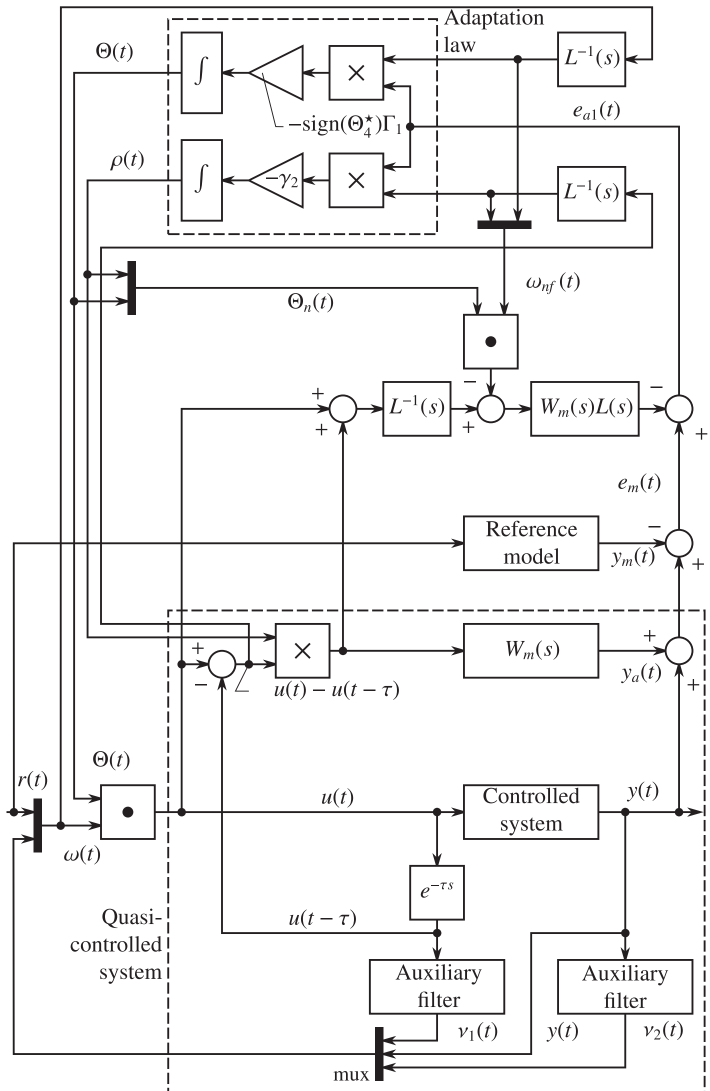


Figure 3: Adaptive control algorithm overall scheme.

controlled system output behavior. This can obviously cause that the adaptation error transient behavior is not satisfactory in terms of the standard MRAC. However, in the non-perfect model following algorithm, the resultant control performance in terms of the actual controlled system output is more important than the transient behavior of the adaptation error.

References

[1] A. BOUBAKIR, S. LABIOD, F. BOUDJEMA and F. PLESTAN: Design and experimentation of a self-tuning pid control applied to the 3dof helicopter. *Archives of Control Sciences*, **23**(3), (2013), 311-331.

[2] S. EVESQUE, A. M. ANNASWAMY, S. NICULESCU and A. P. DOWLING: Adaptive control of a class of time-delay systems. *Journal of Dynamic Systems, Measurement, and Control*, **125**(2), (2003), 186-193.

[3] M. GULAN, M. SALAJ and B. ROHAL'-ILKIV: Application of adaptive multivariable generalized predictive control to a hvac system in real time. *Archives of Control Sciences*, **24**(1), (2014), 67-84.

[4] J.M. LEMOS and M.S. BARAO: A control Lyapunov function approach to adaptive control of hiv-1 infection. *Archives of Control Sciences*, **22**(3), (2012), 273-284.

[5] T. LUDWIG, I. OTTINGER, M. TÁRNÍK and E. MIKLOVIČOVÁ: T1DM subject as a time-delay system: Modeling and adaptive control. *Proc. Int. Conf. on Process Control*, Štrbské Pleso, Slovakia, (2013).

[6] B. MIRKIN and P.-O. GUTMAN: Adaptive following of perturbed plants with input and state delays. *Proc. 2011 9th IEEE Int. Conf. on Control and Automation*, (2011), 865-870.

[7] B. MIRKIN, E. L. MIRKIN and P.-O. GUTMAN: State-feedback adaptive tracking of linear systems with input and state delays. *Int. J. of Adaptive Control and Signal Processing*, **23**(6), (2009), 567-580.

[8] B. MIRKIN, E.L. MIRKIN and P. O GUTMAN: Model reference adaptive control of nonlinear plant with dead time. In *Proc. 47th IEEE Conf. on Decision and Control*, (2008), 1920-1924.

[9] B. M. MIRKIN and P. O. GUTMAN: Output feedback model reference adaptive control for multi-input-multi-output plants with state delay. *Systems & Control Letters*, **54**(10), (2005), 961-972.

[10] S.I. NICULESCU and A. M. ANNASWAMY: An adaptive Smith-controller for time-delay systems with relative degree $n^\star \leqslant 2$. *Systems & Control Letters*, **49**(5), (2003), 347-358.

[11] M. TÁRNÍK: Adaptive posi-cast control for PMSM speed control. *Selected Topics in Modelling and Control*, **8** (2012), 84-87, Bratislava, Slovak University of Technology Press.

[12] M. TÁRNÍK and J. MURGAŠ: Additional adaptive controller for mutual torque ripple minimization in PMSM drive systems. *Proc. of the 18th IFAC World Congress*, Milano, Italy, (2011), 4119-4124.

[13] M. TÁRNÍK, J. MURGAŠ, E. MIKLOVIČOVÁ and L. FARKAS: Adaptive control of time-delayed systems with application for control of glucose concentration in type 1 diabetic patients. *Proc. IFAC Int. Workshop on Adaptation and Learning in Control and Signal Processing*, Caen, France, **11** (2013).

[14] Y. YILDIZ, A. ANNASWAMY, I. V. KOLMANOVSKY and D. YANAKIEV: Adaptive posicast controller for time-delay systems with relative degree $n^* \leqslant 2$. *Automatica*, **46**(2), (2010), 279-289.