Research paper

Effect of icing as a non-structural mass on the variation of natural frequency of a lightweight lattice structure

Wiesław Kowalski[1], Mateusz Richter[2], Katarzyna Tokarczyk[3]

Abstract: This paper analyzes the effect of additional masses for lattice structures on the nature of changes in the natural frequencies of the structure. An attempt to mathematically describe this nature and the scale of the effect with a known thickness of the icing layer was also made. The discussion concerns a structure with a sacred purpose – the Gate of the Third Millennium, located in the Lednickie Fields, in the Kiszkowo Municipality, Gniezno Poviat. The icing of structural bars (frost, rime) is treated as a source of additional masses, although the origin of non-structural mass is of secondary importance for the analysis in question. The analysis was carried out by Finite Element Method (FEM) modeling of the structure, assuming a single-parameter variation of its mass (that is, the additional mass of all elements of the test object varies proportionally to a single parameter, which is the outer surface of the element on which the ice layer is deposited). By solving the vibration eigenproblem for successive models, representing different intensities of icing of the object, the values of successive frequencies and descriptions of the corresponding eigenmodes were determined. The results obtained allow us to formulate a postulate that the possibility of a change in the mass of the analyzed object resulting from icing or other causes should be taken into account in strength analyses, wherein the dynamic properties of the structure play an important role, such as in assessing the susceptibility of the structure to dynamic loads.

Keywords: icing, lattice structure, quasi-truss, transmission tower, wind-induced resonance

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[1]PhD., Eng., University of Agriculture in Krakow, Department of Rural Building, Al. Mickiewicza 24/28, 59-130 Krakow, Poland, e-mail: w.kowalski@ur.krakow.pl, ORCID: 0000-0002-8579-107X
[2]PhD., Eng., University of Agriculture in Krakow, Department of Rural Building, Al. Mickiewicza 24/28, 59-130 Krakow, Poland, e-mail: mateusz.richter@urk.edu.pl, ORCID: 0000-0001-6813-5364
[3]MSc., Eng., University of Agriculture in Krakow,Department of Rural Building, Al. Mickiewicza 24/28, 59-130 Krakow, Poland, e-mail: katarzyna.u.tokarczyk@gmail.com, ORCID: 0000-0002-3668-3873

1. Introduction

In structural theory, the term "dynamic load" has become accepted for certain component loads on structures. We say that an object (a structure or a part of it) is dynamically loaded if the satisfaction of the equilibrium condition for the system of forces acting on this object requires the inclusion of inertia forces in the equilibrium equations (without inertia forces, the equilibrium equations cannot be satisfied). Thus, a dynamically loaded structure is a body "in motion", because one can speak of inertial forces only under such circumstances. Nevertheless, since structures are restrained bodies (structures and their parts have external ties), the motion of a structure should be understood as oscillation around an equilibrium position, or in other words – vibration in a broad sense. Other forms of movement are difficult to imagine for structures, although they can occur with vehicles and other machinery proposed by Urbański et al. [1], Urbański and Richter [2].

The term "resonance" is used when talking about dynamic loads on structures. What we mean here is the relationship in which the frequency at which the "load" applied to the structure appears, to the frequency that characterizes the object dynamically (the frequency of its free vibration). Resonance occurs when the two frequencies have values equal or at least close to each other; the amplitudes of all oscillation parameters (displacements, velocities, accelerations, inertia forces, etc.) increase then, up to a state referred to as failure. It can be destruction or damage to the structure, interference for the operation of machinery installed in the building, harmful effects on the health of its users, sometimes only inconvenience of use (such as unpleasant noise) – still, resonance is always an adverse phenomenon. The designer of a structure subjected to dynamic loads always aims to eliminate resonance. It is therefore necessary to design such a relation of the structure mass to the stiffness of its elements that the free vibration frequencies that result from this relation are as distant as possible from all the frequencies that characterize the dynamic load. Such a process is referred to as "structural tuning" in professional jargon.

In order to complete such "tuning", the designer must know the amplitude-frequency characteristics of the vibration excitation to which the structure will be exposed and the free vibration frequencies of the structure. The extraction of information in the two aforementioned data groups requires special computational procedures, which is often difficult and sometimes fails. The reason for failures may be the difficulty in unambiguously determining the frequency characteristics of the vibration excitation, which is often the case with kinematic excitation of natural (tectonic) origin. For example, successive earthquake events in a given region may have completely different characteristics, which are, after all, dependent on the mechanisms governing how energy is released at the foci of tremors and how seismic waves propagate. These mechanisms cannot be controlled or predicted by humans. The effects of such a situation occurred during the great earthquake in Kobe, Japan, on January 17, 1995, devastating to many structures that were, after all, designed with seismic loads in mind, but with insufficient characteristics in terms of dynamic performance, and failed (Fig. 1).

The reason for errors in proper dynamic "tuning" may also be the difficulty of establishing unambiguous values for the free vibration frequencies of designed (or diagnosed)

Fig. 1. Failure of a flyover on the "Hanshin" expressway after the 1995 Kobe earthquake source: https://www.japantimes.co.jp/opinion/2020/01/16/editorials/learned-enough-1995-kobe-quake/

structures. The free oscillator frequencies depend on the relationship between their mass and stiffness (and their arrangement in the object). Thus, the ambiguity may be due to the variability of both of these quantities (both as to value and location) over the life of the structure. The change in stiffness can result from corrosion processes, changes in the moisture content of materials (e.g., wood) or rheological processes taking place (aging of the material, relaxation in the compression mechanisms of concrete), temperature changes (e.g., during a fire), etc. While a change in the stiffness of a structure usually results from rare and undesirable phenomena that have the character of failures, numerous structures are subject to a change in mass. In addition, these changes occur repeatedly, sometimes even many times a day; a significant part of the variable loads on structures is associated with changes in their mass. Such a situation may be relevant to assessing the vulnerability of a structure to dynamic influences if it is exposed to such influences.

Examples include lattice structures at great heights, such as cellular telephone relay towers, religious buildings (the cross on Giewont – Fig. 2), cable railroad support poles.

Fig. 2. The cross on Giewont in the winter period. Source: https://gazetakrakowska.pl/giewont-zima-olsniewa-krzyz-oblepiony-sniegiem-w-tatrach-o-tej-porze-roku-trzeba-jednak-uwazac-zdjecia/ar/c1-15421329

In a similar situation, we find structures that are significantly higher than those in their neighborhood or located in open areas. In this situation, the structures become particularly exposed to the wind, as there is no natural cover around them. The impact of air movement on the object, in addition to static thrust, can be dynamic here. It is about rhythmic gusts of wind or other effects caused by undisturbed airflow around the object. In such a situation, the structural design process requires an analysis of the structure's susceptibility to wind gusts and, in special situations, it is necessary to analyze the dynamic response of the structure, determined by its dynamic characteristics proposed by He [3], Fekr and McClure [4], Clough and Penzien [5], Fengli et al. [6] which need to be determined. The difficulty in conducting a correct analysis here is due to the periodic variation of mass with the developing icing of the bars (freezing dew), the deposition of rime and sometimes additionally freezing wet snow on the bars of the structure.

Ice deposition on parts of building structures, vehicles and machinery, as a constant phenomenon in cold climate conditions, has been an area of interest to the world of science and technology for many years (Macklin [7], List [8], Makkonen [9]). The problem here is the issue of monitoring the facilities, for early prediction of dangerous conditions associated with excessive ice accumulation proposed by Zaharov [10], Podrezov et al. [11], Lehky et al. [12], Harstveit et al. [13], Lehky and Sabata [14], Vaculik and Rampl [15], preventing excessive development of such phenomena, such as through the use of release coatings (Shigeo et al. [16], Kimura et al. [17]), and developing procedures and calculation techniques for reliable estimation of the extent of icing as an additional load on structures (PN-87/B-02013 [18], ISO 12494 [19], Fikke et al. [20], Xie and Sun [21], Yang et al. [22]). The problem of object icing is of practical importance in regions with cold climates, including mountains, with high humidity, such as in marine conditions (Makkonen [23,24], Gates [25] and Hørjen [26]).

In terms of dynamic issues considered here, in the general case, icing creates two types of problems:

1) it represents a source of additional mass, affecting the change of dynamic characteristics of the structure, and thus interfering with its "susceptibility" to resonance (e.g., due to wind gusts, as already mentioned) proposed by Eliasson and Thorsteins [27] and Clough and Penzien [5],

2) cracking and falling masses of ice are excitation the structure to vibrate. The phenomenon is of particular importance for power lines, where falling ice is a kind of impulse, triggering free vibration of catenary lines, further amplified by gusts of wind which leads to vibration of pylons forced by swaying catenary wires Fekr and McClure [4], Battista et al. [28], Havard and Dyke [29], Kálmán et al. [30], Marzaneh [31], Fengli et al. [6], Chen et al. [32].

In this paper, simulation calculations were carried out to determine the degree of reduction in the natural frequency of a lightweight lattice structure, as its mass increases due to the increasing layer of icing on the structure's bars. The results obtained in the course of the work have cognitive significance. First of all, they are a contribution to the discussion of the susceptibility of structures to wind gusts and the risk of error in assessing them under variable mass conditions. The problem is of particular importance in the design

and technical diagnostics of lightweight lattice structures, for which icing can represent a significant increase in mass in relation to their own weight; especially structures located at high altitudes or in open terrain. The location in a harsh climate results in structures encountering particularly favorable conditions for ice deposition; they are also particularly exposed to wind in such conditions.

2. Materials and methods

2.1. Model specificity

The structure analyzed in the paper is the Gate of the Third Millennium, located in the Lednickie Fields in the Kiszkowo Municipality, Gniezno Poviat. The structure is located in an open area and is not shielded by other structures. Its shape resembles a fish (often referred to as a "fish gate"); therefore it can hardly be called a "tower object". However, it has numerous features typical of quasi-truss tower designs:
- is a structure of considerable size (height 12.5 m; length 38.5 m);
- is largely "saturated" with bars, with a significant lateral area where ice can be deposited (about 170 m^2);
- has a low dead weight (the mass "participating" in the vibration – about 5158 kg), which means that the effect of icing on its percentage increase can be significant;
- is a structural type with low damping (for steel frames and lattices, a critical damping fraction of $\varsigma = 5\%$ or less is frequently being assumed [5, 33–36], so the problem of resonance associated with wind gusts is of particular importance here.

The tower's support structure is made of 952 tubular bars, with a total length of about 1240 m, welded at nodes to steel "balls" stamped from sheet metal. It was designed by Anna Boryska (architecture) and Wiktor Dziembay (construction) in 1997, and is a unique, one-of-a-kind structure. For the sake of clarity in the text and to protect the reader from an excess of information, a detailed description of the characteristics of successive bars and nodes was abandoned. Here, the structural bars are connected by welding to steel sheet "balls" that act as gusset plates (Fig. 4), which is a rigid assembly that does not allow mutual displacement of the bars – so it is a departure from the postulates for lattices and requires, at the stage of creating the numerical model, treating the structure as a spatial frame.

The shape of the "gate-fish" is shown in Fig. 3. A FEM model of the structure was built (using Robot Structural Analysis software), utilizing technical documentation of the facility provided courtesy of the LEDNICA 2000 Community. In doing so, the following assumptions were made:
- Subsequent variants of the model are framework-based, giving freedom to model connections. In the actual structure, the bars were connected at the nodes by welding, which does not provide freedom for rotation and is a deviation from lattice postulates.
- The additional, non-structural inertia of the object, associated with the node plates used, extruded into the shape of a sphere, is taken into account in the model in the form

of concentrated masses (due to the small dimensions of the spheres, an assumption was made about their lack of rotational inertia) at the nodes of the structure.

– The additional, non-structural inertia of the object, associated with structural icing, is included in the model in the form of mass distributed uniformly along the bars (it therefore has rotational inertia).

– In successive variants of the model, the increase in mass resulting from icing, is single-parametric, that is, the additional mass of all elements changes in proportion to a single parameter, which is the external surface area of the element on which the ice layer is deposited.

– The value of specific gravity of icing was adopted based on PN-87/B-02013 [18].

– The flexibility of the subsoil at the foundation level of the analyzed object was neglected.

– The material of the structure works in the linear-elastic range.

– The design is not analyzed to meet ultimate limit states (ULS) and serviceability limit states (SLS).

Fig. 3. Shape of the analyzed structure.
Source: https://tenpoznan.pl/gniezno-zaczyna-sie-lednica-2000/

More than a dozen variants of the model were prepared, corresponding to the system's weight gain due to icing of different intensities. The thickness of the icing layer included in the modeling of the subsequent models was characterized in Table 1.

The thickness of the icing layer was treated here purely formally, with no research as to the extent of the practically achievable thickness of the icing layer. The purpose of the analysis carried out in this paper is to evaluate the nature of the changes in the natural frequencies of the structure as a result of the computational procedure being used, as well as the mathematical description of this nature. It is not the purpose of the analysis to assess the probability of a structural icing load of a certain value. The shape of the FEM model of the structure is shown in Fig. 4–6.

Table 1. Characteristics of bar icing in subsequent structural models

Model number	Thickness of ice layer [cm]	Total weight of the model [kg]	Model number	Thickness of ice layer [cm]	Total weight of the model [kg]	Model number	Thickness of ice layer [cm]	Total weight of the model [kg]
1	0	5158	6	2.5	9761	11	5	17772
2	0.5	5806	7	3	11091	12	6	21930
3	1	6590	8	3.5	12557	13	8	31881
4	1.5	7511	9	4	14159	14	10	44014
5	2	8568	10	4.5	15897	15	20	137384

Fig. 4. Photograph of a typical node ball in a structure
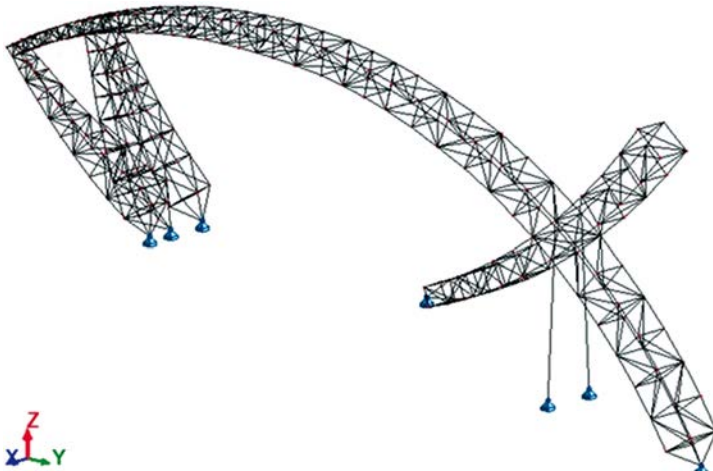
Fig. 5. Model of the analyzed structure and its relation to the axis of the adopted reference system
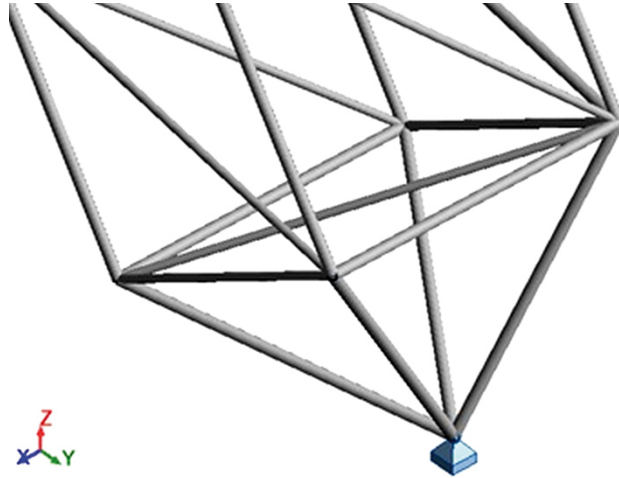
Fig. 6. One of the support nodes of the structure

2.2. Natural vibrations

The theoretical basis for describing the free vibration problem (Clough and Penzien [5]) is most easily presented on the basis of the so-called "displacement method", starting from the free vibration equation of a system (with multiple dynamic degrees of freedom):

$$(2.1) \qquad [M] \cdot \underline{\ddot{y}} + [C] \cdot \underline{\dot{y}} + [K] \cdot \underline{y} = \underline{0}$$

wherein: $[M]$, $[C]$, $[K]$ – quadratic matrices characterizing the dynamic properties of the vibrating system (inertia, damping and stiffness matrix respectively), $\underline{\ddot{y}}$, $\underline{\dot{y}}$, $\underline{y}$ – vectors: acceleration, velocity and displacement (relative) of the oscillating system.

The dynamic eigenproblem, in structural dynamics, boils down to determining the circumstances under which the free vibration equation of a system, lacking an element representing damping, can have a non-zero solution. Thus, the equation (2.1) does not include the damping, and the recipe takes the form:

$$(2.2) \qquad [M] \cdot \underline{\ddot{y}} + [K] \cdot \underline{y} = \underline{0}$$

Assuming a harmonic form of oscillatory motion:

$$(2.3) \qquad \underline{y} = \underline{A} \cdot \sin(\omega \cdot t + \phi) \;\; \rightarrow \;\; \underline{\ddot{y}} = -\omega^2 \cdot \underline{y}$$

wherein: $\underline{A}$ – amplitude of vibration, ω – the circular frequency of the vibration in rad/s; $\omega = 2\pi \cdot f$, f – frequency of the vibration in Hz, φ – phase shift angle, t – time, as a variable in the vibration process.

We get the relationship:

$$(2.4) \qquad -\omega^2 \cdot [M] \cdot \underline{y} + [K] \cdot \underline{y} = \underline{0}$$

By multiplying both sides by the flexibility matrix of the system $[F]$ (left-hand side), we get:

$$(2.5) \qquad -\omega^2 \cdot [F][M] \cdot \underline{y} + [F][K] \cdot \underline{y} = \underline{0}$$

However, since $[F] \cdot [K] = [I]$ we get $\left\{ [F] \cdot [M] - \dfrac{1}{\omega^2} \cdot [I] \right\} \cdot \underline{y} = \underline{0}$ or in another form, taking $\lambda = \dfrac{-1}{\omega^2}$ we get:

$$(2.6) \qquad \{ [F] \cdot [M] - \lambda \cdot [I] \} \cdot \underline{y} = \underline{0}$$

It is a dynamic eigenproblem. The task here comes down to solving a system of homogeneous linear equations. For all variants of the model of the analyzed structure, it is a system of 1,800 equations, which corresponds to the number of 300 nodes with an assigned mass, with six independent degrees of freedom each.

3. Results

For the adopted models of the analyzed structure, the ten lowest natural frequencies were determined successively. The results of the calculations are summarized in Table 2.

Table 2. Ten lowest determined natural frequencies of structure models

Model number	Consecutive, determined natural frequencies of the models [Hz]									
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
1	3.6601	4.3355	5.5489	6.0043	11.170	11.759	13.992	14.268	15.931	16.477
2	3.4487	4.0871	5.2338	5.6632	10.525	11.075	13.184	13.443	15.011	15.522
3	3.2363	3.8364	4.9159	5.3185	9.8767	10.387	12.370	12.613	14.085	14.561
4	3.0311	3.5936	4.6077	4.9840	9.2491	9.7241	11.584	11.810	13.189	13.634
5	2.8378	3.3644	4.3166	4.6680	8.6577	9.0994	10.843	11.053	12.345	12.760
6	2.6587	3.1518	4.0462	4.3743	8.1094	8.5208	10.157	10.352	11.563	11.951
7	2.4943	2.9565	3.7976	4.1043	7.6062	7.9902	9.5271	9.7089	10.846	11.209
8	2.3444	2.7782	3.5705	3.8577	7.1472	7.5066	8.9523	9.1219	10.191	10.532
9	2.2079	2.6160	3.3635	3.6331	6.7296	7.0668	8.4295	8.5880	9.5957	9.9171
10	2.0838	2.4685	3.1752	3.4287	6.3501	6.6674	7.9543	8.1029	9.0544	9.3577
11	1.9710	2.3344	3.0039	3.2428	6.0051	6.3044	7.5223	7.6620	8.5623	8.8493
12	1.7746	2.1010	2.7053	2.9191	5.4047	5.6731	6.7706	6.8949	7.7061	7.9646
13	1.4722	1.7419	2.2450	2.4205	4.4812	4.7026	5.6141	5.7153	6.3890	6.6040
14	1.2533	1.4821	1.9115	2.0596	3.8131	4.0010	4.7775	4.8625	5.4364	5.6198
15	0.7098	0.8382	1.0828	1.1648	2.1574	2.2632	2.7035	2.7500	3.0756	3.1802

Figure 7 summarizes the degrees of reduction of successive natural frequencies Θ_1 due to a change in the mass of the tower's vibrating system caused by 1 cm thick ice. Figure 8 shows the degree of reduction of successive natural frequencies Θ_2 due to a change in the mass of the tower's vibrating system induced by 2 cm thick ice; similarly, in Fig. 9, the parameter Θ_4 describes the change in the mass of the tower's vibrating system induced by 4 cm thick ice. It should be noted here that the determined natural frequencies were related to the corresponding results for model No. 1 in all cases.
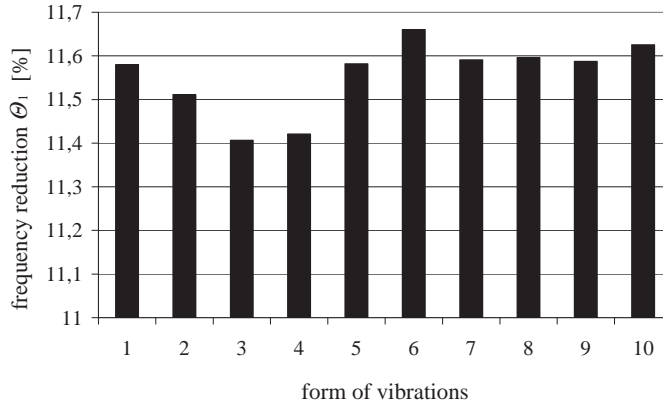
Fig. 7. Degree of reduction of natural frequencies in the tower model Θ_1 with ice thickness of 1 cm in relation to the structure with no icing; for the first form of vibration $\Theta_1 = 11.58\%$
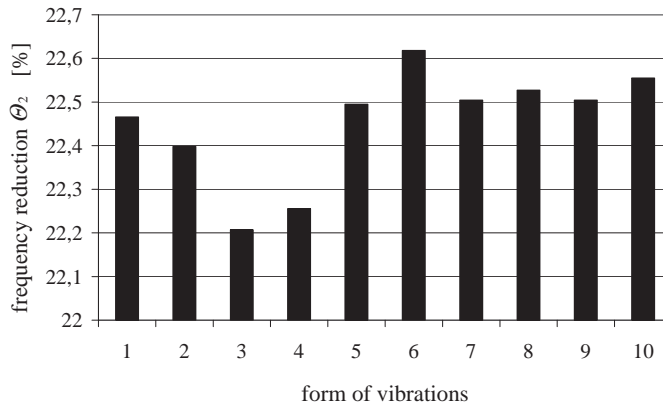
Fig. 8. Degree of reduction of natural frequencies in the tower model Θ_2 with ice thickness of 2 cm in relation to the structure with no icing; for the first form of vibration $\Theta_2 = 22.466\%$

An attempt was made to mathematically describe the relationship between the natural frequency and the mass of the structure, for successive forms of vibration. Due to the peculiarities of this relationship, the results obtained were approximated by a hyperbolic-
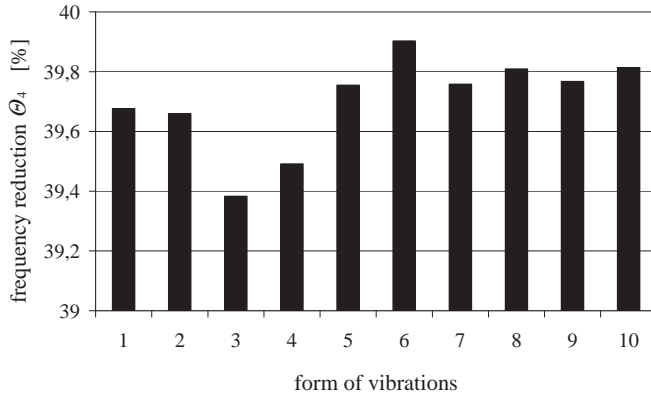
Fig. 9. Degree of reduction of natural frequencies of the tower model Θ_4 with ice thickness of 4 cm in relation to the structure with no icing; for the first form of vibration $\Theta_4 = 39.676\%$

type function:

$$(3.1) \qquad f_N(m) = \frac{A}{m^B}$$

modified, after establishing the dependence of the mass of the oscillating system on the thickness of the icing layer (see Fig. 10) to the form:

$$(3.2) \qquad f_N(d) = \frac{A}{\left(272.5707 \cdot d^2 + 1159.8726 \cdot d + 5158.4821\right)^B}$$

wherein: d – thickness of the ice layer in cm, m – total mass, participating in the vibration of the structure in kg, A, B – coefficients of the model [–].
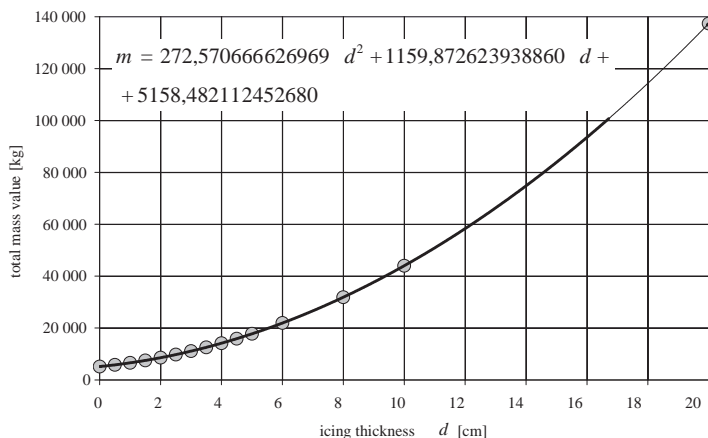
Fig. 10. Dependence of the total weight of the structure on the thickness of the icing layer (determination coefficient [37] $R^2 = 1$). It should be emphasized here that the obtained relationship is inseparable from the assumption of a one-parameter variation of mass, proportional to the lateral area of the bars

The hyperbolic type relationship has the important property that an increase in the thickness of the ice layer is always accompanied by a decrease in the value of the frequency (at infinity – an asymptotic decrease to zero), which corresponds to the physical interpretation of the phenomenon. It is possible to make a polynomial-typeapproximation, using simple polynomials, of the third or fourth order, with very good results, but the best-adapted polynomials are often increasing functions on certain sections, which corresponds to the situation when an increase in mass causes an increase in the natural frequency. Moreover, many functions of this type take negative values in certain ranges (yet they denote frequency, a non-negative quantity). Mathematically, such a description is correct, but it is absurd on physical grounds. For presentation purposes, an example of this situation is shown in Fig. 11–13, for the basic form of vibration.

$$f_1 = 0{,}00002298 \; d^4 - 0{,}00162382 \; d^3 + 0{,}04280956 \; d^2 + {}$$
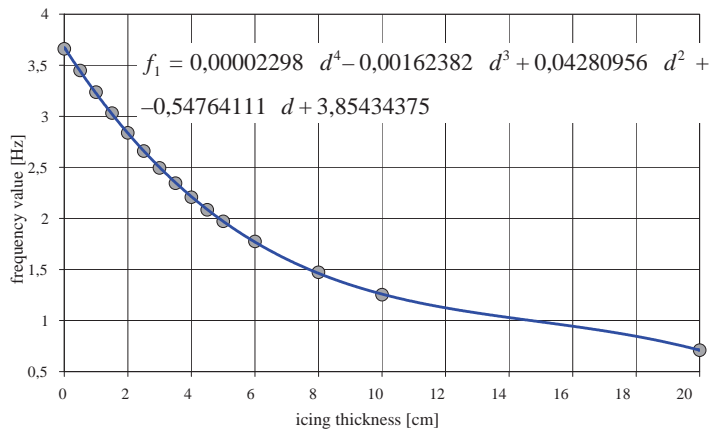$$-0{,}54764111 \; d + 3{,}85434375$$

Fig. 11. The fundamental natural frequency of the model and its, apparently correct, polynomial dependence on the thickness of the icing layer; $R^2 = 0.999924689972$
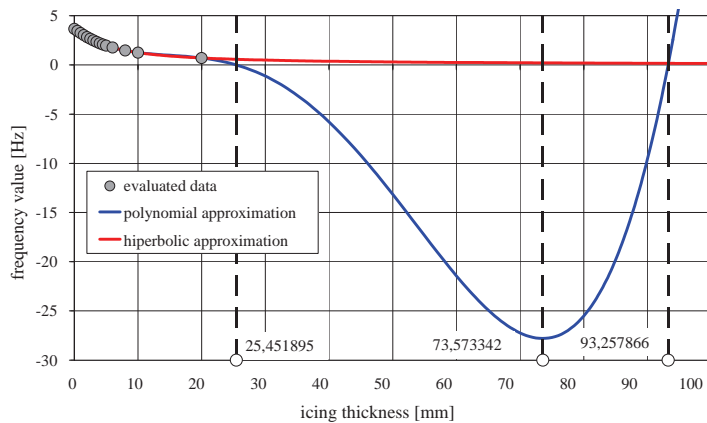

Fig. 12. Comparison of the waveforms of functions of different types, describing the fundamental natural frequency for large values of the argument (icing layer thickness)
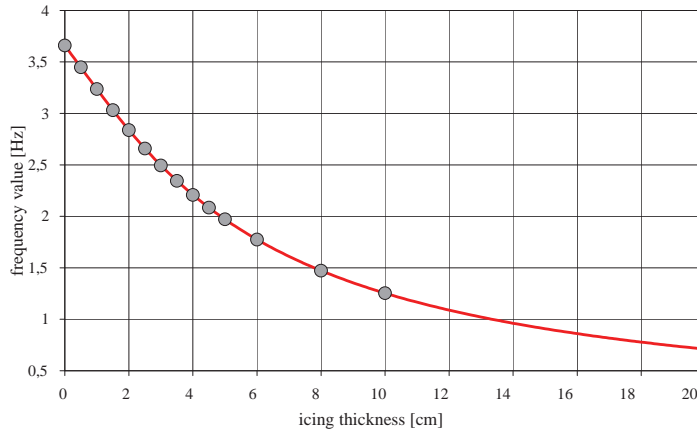
Fig. 13. The fundamental natural frequency of the model and its **hyperbolic** dependence
on the thickness of the ice layer (also marked with a red line in Fig. 12)

The dependence of frequency on icing thickness, shown in Fig. 13, can be expressed
in the form:

$$f_1 = \frac{261.807320462682}{(272.570666626969 \cdot d^2 + 1159.872623938860 \cdot d + 5158.482112452680)^{0.499608384553}}$$

The foregoing figures show the strongly nonlinear dependence of the structure's fun-
damental natural frequency on the thickness of the ice layer. However, for small values of
icing thickness, the foregoing relationship can be expressed, with fairly good (technical)
accuracy, through a linear relationship (see Fig. 14), which simplifies calculations in en-
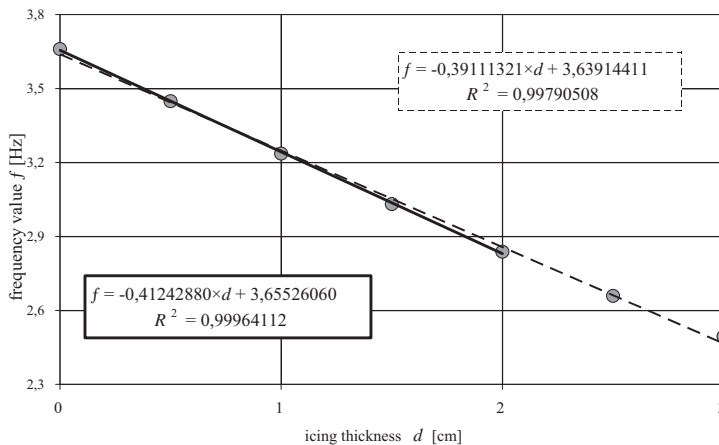

Fig. 14. Linear description of the dependence of the structure's fundamental natural frequency on the
thickness of the ice layer in the 0–2 cm and 0–3 cm ranges; in both cases, the high degree of linear
function adjustment to the determined frequency values is noteworthy

gineering analyses. Such an approach, however, requires a case-by-case assessment of the accuracy of the calculations, which may be questioned.

Table 3 summarizes the values of parameters A and B for equation (3.2), which describes the relationship of successive, determined natural frequencies to the thickness of the icing layer. It should be clarified here that the parameters in the table are not assigned units which does not mean that they are dimensionless quantities. To avoid unnecessary complications in this regard, the authors propose treating the determined parameter values as empirical, inseparable from the form of formula (3.2), and the assumption made that the thickness of the icing layer in the foregoing formula is expressed in centimeters and the determined frequency – in Hertz.

Table 3. The determined values of parameters A and B for equation (3.2)

Number of the next form of vibration	A [–]	B [–]
1	261.807320462682	0.499608384553
2	313.621640382496	0.500788821529
3	391.706078778539	0.497811862192
4	431.061336929762	0.499762427381
5	809.041522340077	0.501045235776
6	858.524674036729	0.502096539412
7	1011.97810387954	0.500893556092
8	1039.29857357226	0.501726014591
9	1155.00290027442	0.501169891307
10	1194.19423319644	0.501193104112

4. Final conclusions

Subsequent determined natural frequencies "increase" quite slowly, which is a characteristic of "lattice" slender structures. For example, in case of a model without additional mass, the tenth natural frequency is only 4.5 times higher than the base frequency. Due to structural symmetry (slight "deviations") – the values of natural frequencies are arranged in groups of two or three, with similar values.

The relationship between the thickness of the icing and the change in the natural frequency of the "gate" is strongly nonlinear, for each of the ten lowest frequencies determined. This nonlinearity can be expressed, with satisfactory accuracy, through the shape of a hyperbola. For small thicknesses of the ice layer, of the order of a few centimeters, the effect of icing on the change in the natural frequency of the "gate" can be treated as linear, with technically satisfactory accuracy.

Icing significantly alters the dynamic characteristics of the analyzed structure; with a change in the thickness of the ice layer from zero to 2 cm, which does not seem an exorbitant, unrealistic value, the subsequent natural frequencies are reduced by at least 22%. On the other hand, icing with a thickness of 4 cm results in a reduction of all determined natural frequencies by at least 39%.

The conclusion above gives cause for concern about the situation of significant icing, whether the analyzed "fish gate" will not become susceptible to wind gusts, especially since it is a structural type with low internal damping. Nevertheless, a detailed analysis of this problem is beyond the scope of this paper.

All conclusions, formulated above as a result of conducted analysis, refer to structure under consideration only and should not be extrapolated to another cases.

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Wpływ oblodzenia jako masy niekonstrukcyjnej na zmianę częstotliwości drgań własnych lekkiej konstrukcji typu kratownicowego

Streszczenie:

W pracy dokonano analizy wpływu dodatkowych mas dla konstrukcji typu kratownicowego na charakter zmian częstotliwości drgań własnych tej konstrukcji. Przeprowadzono też próbę matematycznego opisu tego charakteru oraz skali wpływu przy znanej grubości warstwy oblodzenia. Rozważania dotyczą budowli o przeznaczeniu sakralnym – Bramy Trzeciego Tysiąclecia, zlokalizowanej na Polach Lednickich w gminie Kiszkowo w powiecie gnieźnieńskim. Jako źródło dodatkowych mas traktowane jest oblodzenie prętów konstrukcyjnych (szron, sadź), jakkolwiek pochodzenie masy niekonstrukcyjnej ma dla przedmiotowej analizy znaczenie drugorzędne. Analizy dokonano w drodze modelowania Metodą Elementów Skończonych (MES) konstrukcji, przyjmując założenie o jednoparametrycznej zmienności jej masy (to znaczy, że dodatkowa masa wszystkich elementów badanego obiektu zmienia się proporcjonalnie do jednego parametru, którym jest powierzchnia zewnętrzna elementu, na której odkłada się warstwa lodu). Rozwiązując zagadnienie własne drgań dla kolejnych modeli, reprezentujących różne intensywności oblodzenia obiektu, wyznaczono wartości kolejnych częstotliwości i opisy odpowiadających im postaci drgań własnych. I tak, przyrost grubości warstwy lodu na powierzchniach, od 0 do 1 cm spowodował redukcję wszystkich wyznaczonych (dziesięciu podstawowych) częstotliwości drgań własnych o co najmniej 11%. Grubość oblodzenia 2 cm wpływa na redukcję częstotliwości, jak wyżej, o ok. 22% zaś 4 cm warstwa oblodzenia oznacza redukcję przedmiotowych częstotliwości powyżej 39%. Są to istotne wartości, z punktu widzenia zastosowań technicznych. Uzyskane wyniki pozwalają na sformułowanie postulatu, aby w analizach wytrzymałościowych, w których istotną rolę odgrywają właściwości dynamiczne konstrukcji, np. w ocenie podatności konstrukcji na obciążenia dynamiczne, uwzględniana była możliwość zmiany masy analizowanego obiektu wynikająca z oblodzenia lub z innych przyczyn.