

Sound Transmission Through a Thin Plate with Shaped Frequency Response

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Thin plates, in the form of individual panels or whole device casings, often separate the noise source from its recipients. It would be very desirable if the panels could effectively block the sound transmission preventing noise from further propagation. This is especially challenging to achieve at low frequencies. A promising approach, intensively developed in the recent years, is to employ active control methods by adding sensors and actuators, and running a control algorithm. However, if the noise is narrow-band, an alternative passive solution originally developed by the authors can be applied. It is based on appropriately located passive elements which can be used to alter the frequency response of the vibrating structure thus improving its sound insulation properties. Such an approach is referred to as the frequency response shaping method. The purpose of this paper is to further develop this method and apply it to a device casing panel. The efficiency of the method is evaluated by simulation and real experiments. Appropriate cost functions and mathematical models are formulated and used to optimise the arrangement of passive elements mounted to the plate, enhancing its sound insulation properties at the given frequency range. The results are reported, and advantages and limits of the method are pointed out and discussed.

Keywords: structural control; frequency response shaping; vibrating plate; optimisation; modelling.

1. Introduction

Excessive noise affects life of many people. In the case of high intensity and prolonged exposure, it can be dangerous to human health. Otherwise, even if the noise does not represent a direct threat, it can obstruct work or leisure. Hence, people often seek a way to reduce it. Devices and machinery are common sources of noise in everyday life and they are often separated from the recipients with thin plates, in the form of individual panels or whole device casings. It would be very desirable if the panels could effectively block the sound transmission preventing noise from further propagation. However, this is challenging to achieve, especially at low frequencies. Sound transmission and radiation of thin plates have been studied by many researchers (KAUR, KHANNA, 2018; KLANNER, ELLERMANN, 2018; RDZANEK, 2018; SZEMELA *et al.*, 2018; ZAWIESKA, RDZANEK, 2014).

Passive sound insulating materials are commonly used to limit the sound transmission. However, they are often ineffective at low frequencies or are inapplicable due to the increase of size and weight. A promising approach, intensively developed in the recent years, is to employ active control methods by adding sensors and actuators, and running a control algorithm (JUNG *et al.*, 2019; LENIOWSKA, MAZAN, 2015; MISOL *et al.*, 2018; MORZYŃSKI, SZCZEPAŃSKI, 2018). Active methods are especially efficient at low frequencies, where passive sound insulations fail. If the active methods are applied according to the active casing approach, they also provide global noise reduction (in the whole room or environment) (MAZUR *et al.*, 2019; WIORA *et al.*, 2017). However, in applications where external source of energy is completely unavailable and the noise is narrowband, an alternative passive solution originally developed by the authors can be applied. It is based on appropriately located passive elements which can be

used to alter the frequency response of the vibrating structure, thus improving its sound insulation properties at a targeted frequency band. Such an approach is referred to as the frequency response shaping method (WRONA, PAWELCZYK, 2016b; 2016c).

For device casings, it means mounting several additional ribs and/or masses to the panels' surfaces at locations followed from an optimisation process. The general rules are known – additional masses lower the natural frequencies of the plate, whereas stiffening ribs elevate them. However, before the research presented by the authors, presence of additional masses and stiffeners has not been analysed and used together, especially for shaping the frequency response according to precisely defined demands. The shaping of the frequency response can highly enhance passive attenuation for certain frequency bands. Moreover, if active control is considered, sensors and actuators arrangement can be optimised together with passive masses and ribs, improving the structure control susceptibility. The limits are related mainly to the maximum dimensions and mass of the created structure.

The purpose of this paper is to further develop this method and to apply it to a device casing panel. The aim is to enhance its sound insulation properties at a given frequency range. The novelty of the presented research is the analysis of sound insulation introduced solely by the passive control method in such a way that it can be easily compared to other noise reduction means. The efficiency of the method is evaluated by simulations and real experiments. Appropriate cost functions and mathematical models are formulated and used to optimise the arrangement of passive elements mounted to the plate. The results are reported, and advantages and limits of the method are pointed out and discussed.

2. Laboratory setup

The presented study employs a rigid laboratory casing, previously used by the authors to investigate active noise reduction methods based on the active casing approach (MAZUR *et al.*, 2018; WRONA, PAWELCZYK, 2015). The casing has a heavy cubic frame made of 3 mm thick welded steel profiles. The high rigidity of the frame results in its resonance frequencies to be far above frequencies of the noise considered. The bottom of the casing is vibrationally and acoustically insulated. In general, all sides of the casing can be made of single or double panels. However, to facilitate the investigation of the considered passive control concept, a single thin plate (casing panel) is investigated in the present research. Hence, a steel thin plate is mounted to the front of the casing, while the remaining panels of the casing are passively insulated with sound-absorbing materials. The utilised casing is depicted in Fig. 1.

The tested front panel is attached to the structure by 20 screws embedded in the frame and clamped with an additional steel square frame. In this stage of the research, a loudspeaker placed on the casing floor is used as the noise source. It allows for creating an environment more suitable for the research than real operating devices which are considered in other research. The loudspeaker is used to generate a tonal noise or a random broadband noise to measure the sound transmission through the thin steel plate, with or without additional loading attached to its surface.

To shape the frequency response of the investigated plate, and to enhance its sound insulation for a specific frequency band, additional masses are attached to the plate surface (in this research additional ribs are not employed. However, they can be included to further ex-

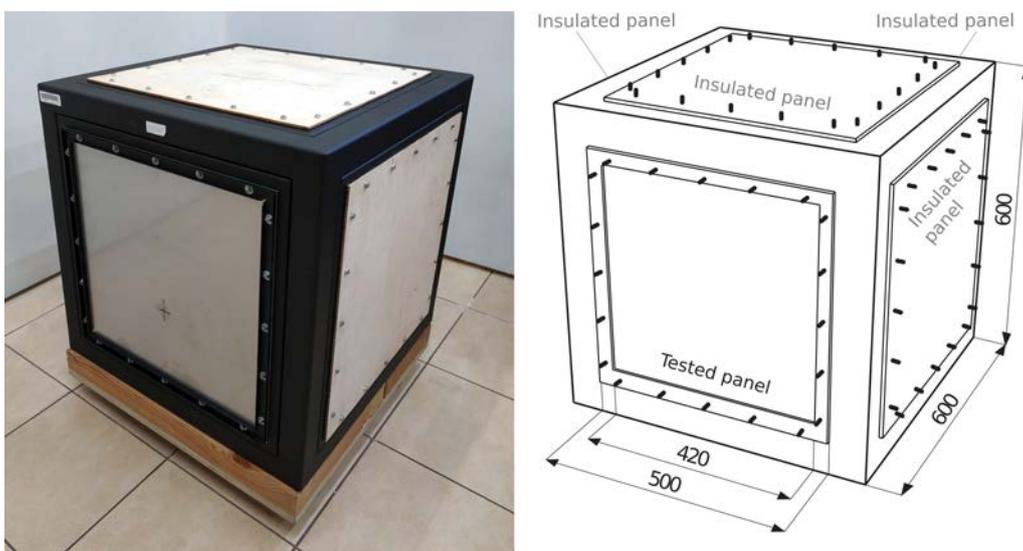


Fig. 1. Photo and a scheme of the rigid laboratory casing. All dimensions are given in [mm].

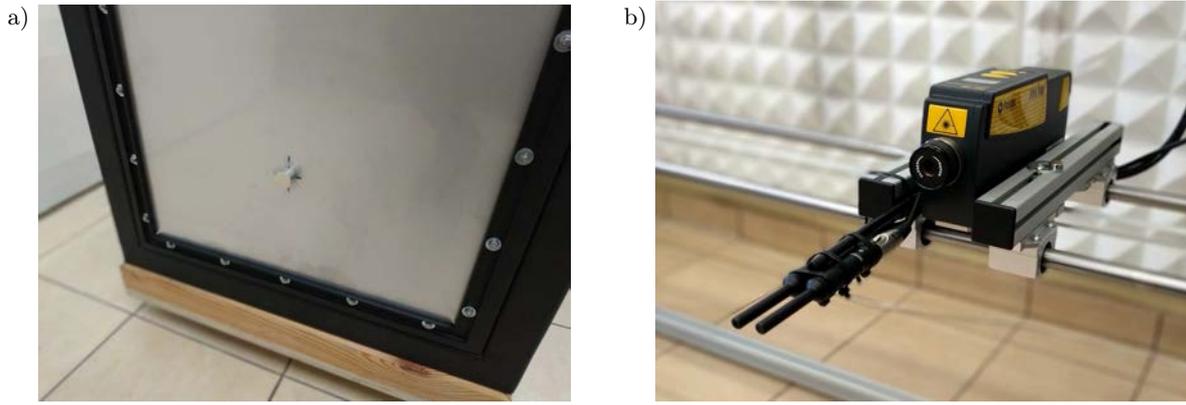


Fig. 2. Photo of the neodymium magnets attached to the plate surface as an additional mass and one of a carriage with a laser vibrometer and microphones: a) neodymium magnets attached to the plate, b) automatic positioning system carriage.

tend the method’s capabilities (WRONA, PAWELCZYK, 2016b; 2016c)). Neodymium magnets are used in the role of the additional masses, due to their facilitated mounting to the steel plate. Several neodymium magnets forming a single additional mass attached to the plate surface are visible in Fig. 2a.

Sound transmission through the thin plate is measured with Beyerdynamic MM-1 measurement microphones. One microphone is placed inside the casing, in front of the loudspeaker to provide measurements of the generated noise. Two other microphones are used to measure sound outside the casing. They are mounted to the carriage of the automatic positioning system visible in Fig. 2b (the positioning system can move the carriage in two axes – horizontal and vertical – precisely). A laser vibrometer Polytec PDV-100 is also mounted to the carriage. In the presented research, the vibrometer was used to provide the measurements of the plate vibrations at 400 points, organised in a square-shaped measurement grid (20 by 20 points, separated by a distance of 20 mm from each other and covering the entire surface of the casing panel). During the same measurement procedure, the microphones provided the measurements of the sound transmitted through the plate. Joined together, the obtained data provide a valuable insight into the behaviour of the considered vibroacoustic system.

3. Frequency response shaping method

An ability to shape frequency response of a vibrating plate according to precisely defined demands has a very high practical potential. It can be used to improve acoustic radiation of the plate for required frequencies or to enhance acoustic insulation of individual barriers or even whole device casings. The method is based on mounting several additional elements (masses and/or ribs) to the plate surface at locations followed from an optimisation process. In

this section, the frequency response shaping method is briefly recalled. However, a detailed derivation and the study of the method are presented in previous publications of the authors (WRONA, PAWELCZYK, 2016b; 2016c).

3.1. Mathematical modelling

The response of a physical plant excited to vibrations is determined by its mechanical structure. Even slight modifications of the structure may strongly affect its frequency characteristics. If the results of the structure modification can be accurately predicted by a model, the process can be inverted – an optimisation problem can be formulated where a specific structural modification is searched for, looking for a solution which provides a desired modification of the frequency response. However, the employed model has to be accurate.

A mathematical model employed to reflect the steel plate considered in this research is based on the Mindlin plate theory (MINDLIN, 1951; RAO, 2007). Free vibrations of the isotropic Mindlin plate are governed by the equations:

$$\begin{aligned}
 & \frac{\rho h^3}{12} \frac{\partial^2 \Theta_x}{\partial t^2} - \frac{D}{2} \left[(1-\nu) \left(\frac{\partial^2 \Theta_x}{\partial x^2} + \frac{\partial^2 \Theta_x}{\partial y^2} \right) \right. \\
 & \quad \left. + (1+\nu) \left(\frac{\partial^2 \Theta_x}{\partial x^2} + \frac{\partial^2 \Theta_y}{\partial x \partial y} \right) \right] + \kappa h G \left(\frac{\partial w}{\partial x} + \Theta_x \right) = 0, \\
 & \frac{\rho h^3}{12} \frac{\partial^2 \Theta_y}{\partial t^2} - \frac{D}{2} \left[(1-\nu) \left(\frac{\partial^2 \Theta_y}{\partial x^2} + \frac{\partial^2 \Theta_y}{\partial y^2} \right) \right. \\
 & \quad \left. + (1+\nu) \left(\frac{\partial^2 \Theta_y}{\partial y^2} + \frac{\partial^2 \Theta_x}{\partial x \partial y} \right) \right] + \kappa h G \left(\frac{\partial w}{\partial y} + \Theta_y \right) = 0, \\
 & \rho h \frac{\partial^2 w}{\partial t^2} - \kappa h G \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial \Theta_x}{\partial x} + \frac{\partial \Theta_y}{\partial y} \right) = 0,
 \end{aligned} \tag{1}$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (2)$$

In Eqs (1) and (2) function $w(x, y, t)$ denotes the displacement of the plate from the reference state to the z -direction at time $t > 0$ and position (x, y) ; $\Theta_x(x, y, t)$ and $\Theta_y(x, y, t)$ denote the rotations of a transverse normal at position (x, y) in the x and y directions, respectively; D is the flexural rigidity; E is the Young's modulus; ν is the Poisson's ratio; ρ is the mass density of the plate material; h is the plate thickness; G is the shear modulus; κ is the shear coefficient.

The model includes the impact of actuators' masses by making the mass density of the plate ρ a function dependent on (x, y) coordinates. Visualisation in an isometric projection of a rectangular plate with additional elements has been given in Fig. 3. The model is solved utilising the Rayleigh-Ritz assumed mode-shape method. Characteristic orthogonal polynomials having the property of Timoshenko beam functions are used as eigen-functions satisfying edge constraints (KIM *et al.*, 2012). Due to real (and hence imperfect) mountings of each of the casing walls, boundary conditions elastically restrained against both rotation and translation are employed. It is noteworthy that the spring constants describing boundary conditions of the walls cannot be measured or calculated directly. Therefore, for the purpose of fitting the model to the behaviour of real vibrating structure, an optimisation algorithm is used to identify them. The process of identification based on experimental data is described in detail in (WRONA, PAWELCZYK, 2016a).

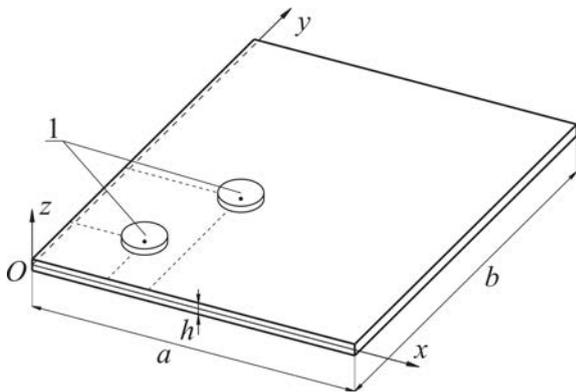


Fig. 3. Visualisation in an isometric projection of a rectangular plate with additional masses (1) bonded to its surface.

The model has been derived and described in detail in (WRONA, PAWELCZYK, 2016b). It is also verified for the considered plant in Sec. 4, where experimentally measured and theoretically predicted natural frequencies are compared (cf. Table 1).

Table 1. The comparison of the eigenfrequencies obtained with experimental measurements and theoretical calculations (unloaded 1 mm thick steel plate).

Mode number	Modeshape	Eigenfrequency (model) [Hz]	Eigenfrequency (experiment) [Hz]
1	11	45	52
2	12	91	88
3	21	93	95
4	22	136	134
5	13	165	157
6	31	168	162
7	23	208	202
8	32	210	204
9	14	263	244
10	41	269	249
11	33	280	274
12	24	305	292
13	42	310	296

3.2. Optimisation process

The search space followed from the frequency response shaping problem is very complicated. An efficient algorithm has to be chosen to find a solution satisfying the defined demands. Evolutionary algorithms have proven to be a versatile and effective technique for solving nonlinear optimisation problems with multiple optima (GOLBERG, 1989). However, they usually require evaluation of numerous solutions resulting in high computational cost. To mitigate this drawback, memetic algorithms can be utilised, which are hybrid forms of population based approach coupled with separate individual learning. Memetic algorithms combine advantages of a global search, like for evolutionary algorithms, and local refinement procedures, which enhance convergence to the local optima (NERI *et al.*, 2012). Because of complementary properties, they are particularly useful in solving complex multiparameter optimisation problems. Hence, a memetic algorithm has been employed to perform the optimisation in the presented research.

The optimisation variables are the coordinates of additional masses on the plate surface. The cost function J is a measure of discrepancy between the desired and actual frequency responses of the plate. Various approaches can be utilised to define the cost function. In the presented research, the goal is to enhance the sound insulation of the plate for a given frequency band, hence the cost function is defined on the basis of natural frequencies, ω_i , where i stands for the eigenmode number. Assuming that the centre of the targeted band is denoted as ω_d , a cost function can be

formulated that expresses the aim to keep natural frequencies as far as possible from the given frequency ω_d :

$$J = -\min_i |\omega_i - \omega_d|, \quad i \in \{1, 2, \dots, N\}, \quad (3)$$

where N is the number of considered eigenmodes.

More forms of particular cost functions are presented in (WRONA, PAWELCZYK, 2016c), where optimisation results for different scenarios are presented and discussed.

4. Sound insulation enhancement of the thin plate

The aim of the research is to enhance the sound insulation of the casing panel for a given frequency band using passive control. A panel used for the experiment is a 1 mm thick steel plate. To achieve the goal, the frequency response shaping method is used, hence, a mathematical model of the panel has to be formulated. While using the model described in Subsec. 3.1, the following parameters have been adopted:

$$\begin{aligned} a &= 0.420 \text{ m}, & b &= 0.420 \text{ m}, & h &= 0.001 \text{ m}, \\ E &= 200 \text{ GPa}, & G &= 76.9 \text{ GPa}, \\ \rho &= 7850 \text{ kg/m}^3, & \nu &= 0.3, & \kappa &= 5/6, \end{aligned}$$

where a and b denote the width and height of the plate, respectively, and they are defined by corresponding dimensions of the structure depicted in Fig. 1.

The accuracy of the model is evaluated by the comparison of the eigenfrequencies obtained with experimental measurements and theoretical calculations (cf. Table 1). The modeshapes of an unloaded plate are regular, hence they can be identified by the number of the nodal lines reduced by one, parallel to y and x directions, respectively. The accuracy of fitting the model to the experimental data is not perfect. However, it is satisfactory for the purpose of the frequency response shaping.

In the role of optimised additional masses, neodymium magnets are used, as described in Sec. 2. Their total mass allowed is equal to 120 g. However, there are multiple pieces of different sizes, hence any mass

in the range up to 120 g can be obtained (adopting accuracy of 2 g or better). It is assumed that the magnet's size is small enough comparing to the dimensions of the plate and that the magnetic force holding the magnets together is strong enough to consider them as a concentrated mass (the rotational inertia is also neglected).

For the performed experiment, it has been assumed that the narrowband noise source has the most of its acoustic energy concentrated in a frequency band around the frequency of 162 Hz. It is one of the worst cases, due to the fact that for this frequency the preliminary experiments show the resonance. Hence, the aim is to move the natural frequencies of the plate as far as possible from the targeted frequency band, therefore enhancing the sound insulation of the plate for the given frequency band. The adopted cost function has a form given in Eq. (3), assuming the centre of the targeted frequency band $\omega_d = 162$ Hz. In the optimisation process presented, only a single concentrated mass location has been considered (maximum mass allowed was equal to 120 g). If the optimisation results were unsatisfactory, more additional elements could be allowed in the optimisation. The memetic algorithm has been employed, as described in Subsec. 3.2.

The results of the optimisation are presented in Fig. 4. The algorithm has placed the additional mass at location ($x = 0.210$ m; $y = 0.143$ m), of maximum allowed mass $m = 120$ g. It follows from the analysis of Fig. 4 that the natural frequencies of the plate have moved significantly away from the $\omega_d = 162$ Hz with a satisfactory margin, allowing some fluctuations of the assumed noise frequency.

4.1. Laboratory experiment

The solution obtained with the frequency response shaping method has been practically implemented. The laboratory setup described in Sec. 2 has been used for this purpose. The loudspeaker placed inside the rigid casing excited the panel with a tonal or broadband noise. Employing the automatic positioning system with the vibrometer and microphones, the vibrations of the plate and the transmitted sound have been

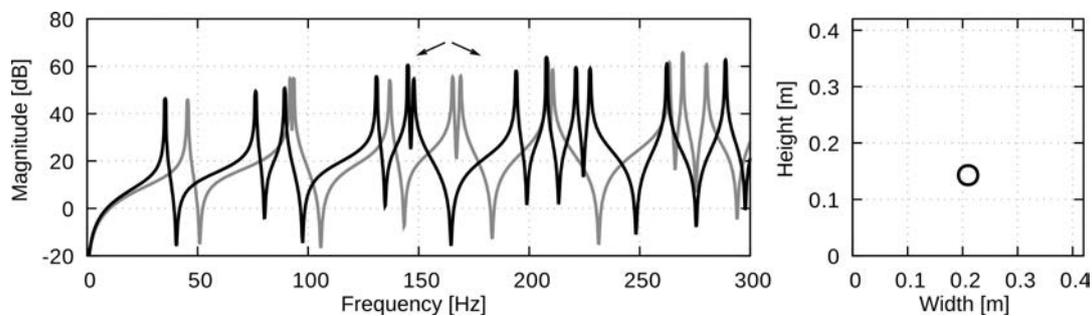


Fig. 4. Results of the optimisation of cost function (3) for a single additional mass. The plots depict the frequency response of the plate (theoretical calculations; black line – plate with the element; gray line – unloaded plate) and visualisation of the additional element mounted on the plate (circle – an additional masses).

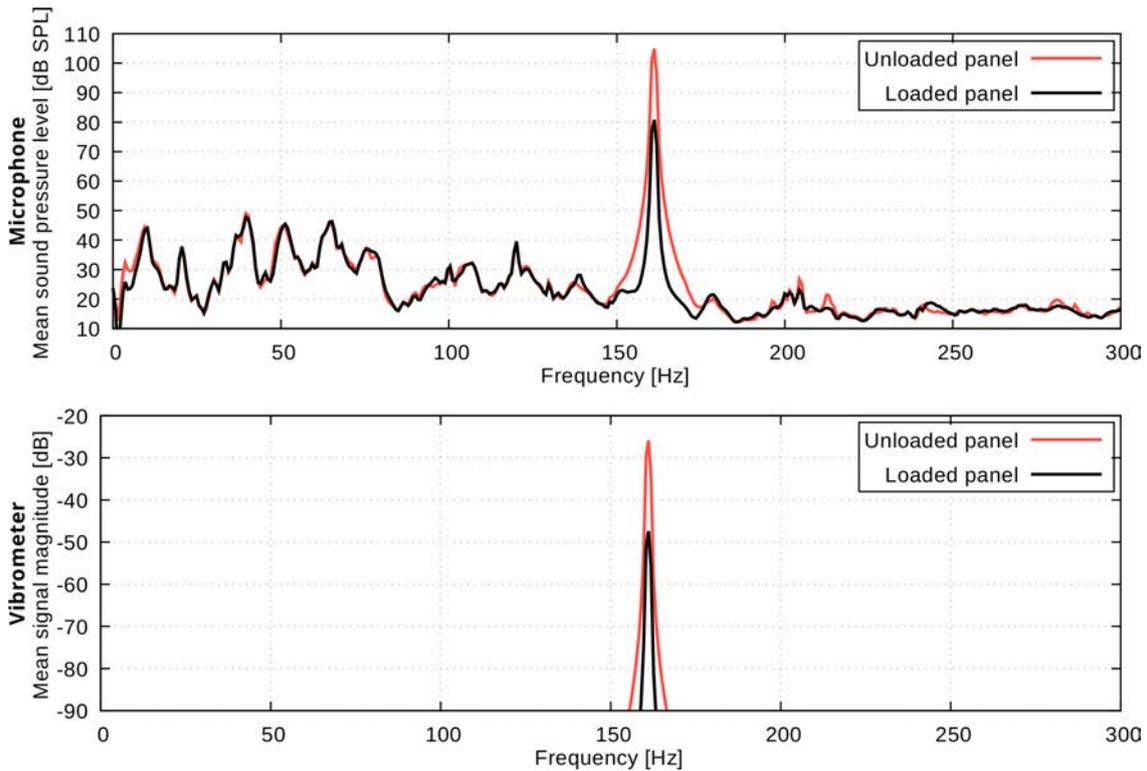


Fig. 5. Experimentally measured mean sound pressure level of tonal noise (162 Hz) transmitted through the plate (top plot) and the mean vibrations of the plate (bottom plot). The mean frequency characteristics have been calculated as averaged characteristics of signals measured over the measurement grid (400 points).

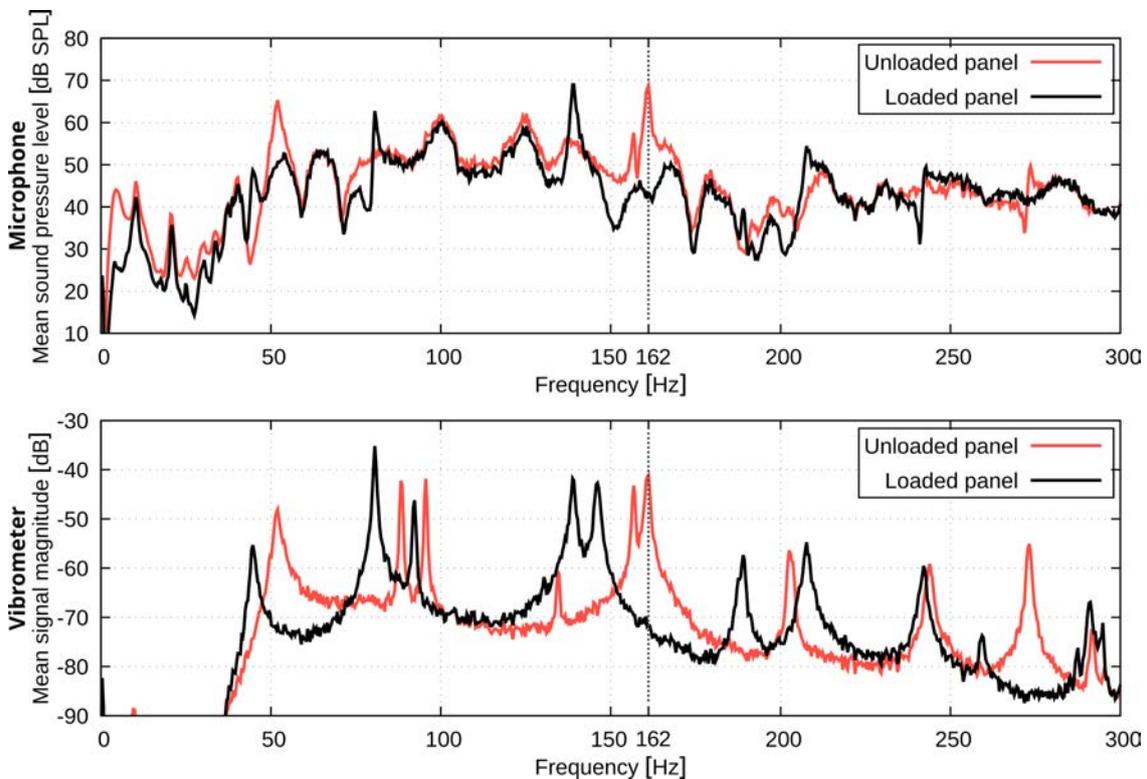


Fig. 6. Experimentally measured mean sound pressure level of broadband noise transmitted through the plate (top plot) and the mean vibrations of the plate (bottom plot). The mean frequency characteristics have been calculated as averaged characteristics of signals measured over the measurement grid (400 points).

measured. The measurements have been done for both unloaded plate and the loaded plate with the frequency response shaping solution.

Figure 5 presents mean sound pressure level of tonal noise (162 Hz) transmitted through the tested plate (with and without the additional mass). It follows from an analysis of Fig. 5 that the sound insulation in the considered frequency band has been enhanced by more than 20 dB using the passive control method. It is a very high level of noise reduction considering the fact that only a single mass has been attached to the plate.

To better notice the effect of the frequency shaping method, the results of a random broadband noise excitation have been presented in Fig. 6. Figure 6 shows that the natural resonances have been moved away from the targeted frequency band of the centre frequency $\omega_d = 162$ Hz. Such experimental measurements are fully consistent with theoretical expectations. It is noteworthy that the mean sound pressure level measurement results presented in Figs 5 and 6 are an average of measurements done in 400 points, confirming that the observed noise reduction is a reduction of total noise emission (not just a change of spatial distribution pattern). Moreover, it is also noteworthy that if larger additional masses were allowed, the width of the obtained frequency band with enhanced sound insulation properties could be even wider.

5. Conclusions

The purpose of this paper was to further develop the frequency response shaping method and apply it to a device casing panel. The efficiency of the proposed method has been evaluated by simulation and real experiments. Appropriate cost function and mathematical model have been formulated, verified, and used to optimise the location of a passive mass mounted to the plate, enhancing its sound insulation properties at the given frequency range.

The obtained results show that the considered method can be effectively applied to enhance sound insulation of a plate or casing panels for a narrowband noise. The proposed passive control approach does not need any source of energy and the cost of the structural modification is marginal in most cases. It can be a very convenient noise solution if the noise is narrowband and rather stationary (some frequency fluctuations are allowed). Many noise sources in real life meet these requirements. The limits are related mainly to the maximum dimensions and mass of the created structure.

However, it is worth emphasising that the method does not attenuate or eliminate the resonances, it moves them in a desired manner. Therefore, for non stationary or broadband noises, a more advanced system should be considered, possibly including an adap-

tation mechanism, e.g. an active control system. Nevertheless, the proposed method can be an interesting option when considering available noise solutions for particular applications, being either a stand alone or a complementing solution.

The main novelty of the presented paper is the implementation and evaluation of the proposed method for sound insulation enhancement of a plate (noise barrier) composed in a real structure (formerly, only a vibrational frequency response has been considered). The analysis of the sound insulation introduced solely by the passive control method is done in such a way that it can be easily compared to other noise reduction means, including active control. The presented results also confirm a high practical potential of the proposed method.

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