



QUANTITATIVE ASSESSMENT OF CONSTRUCTION RISK

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Abstract: Construction risk assessment is the final and decisive stage of risk analysis. When highly changeable conditions of works execution are predicted, risk should be evaluated in the favorable, moderate, and difficult random conditions of construction. Given the random conditions, the schedule and cost estimate of the construction are developed. Based on these values, the risk of final deadline delay and the risk of total cost increase of construction completion are calculated. Next, the charts of the risks are elaborated. Risk changes are shown in the charts and are analyzed in the range [1, 0].

Keywords: risk conditions, scheduling, cost estimating, risk assessment, risk charts

1. INTRODUCTION

In practice, a construction² risk often has to be scrutinized because of the influence of random events over the course and results of works³ execution. Construction is the main stage of construction project⁴. There are a number of publications on different methods of construction risk analysis. The evolution of such concepts, an overview of research and applications pertaining to reliability of construction production, the use of resources, robust itineraries, and contingency of time and cost has been presented in publication [9]. Authors have described areas of management advisory systems in relation to the cycle of risk analysis. They also point out emerging trends which standardize the investment process management, including risk analysis. The investigation of risk management

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² Construction – the building of structures and facilities

³ Construction work – an activity involving mental or physical effort done in order to build structures and facilities

⁴ Construction project – individual or collaborative activities planned and designed to build structures and facilities; Life cycle of construction project: planning phase, design phase, construction phase, operation phase;

in small projects in Singapore in terms of barriers and impact of risk management on project performance has been presented in the article [2]. Based on the results of a questionnaire survey, the authors pointed to a positive correlation between risk management and the improvement in quality, cost, and schedule performance of small construction projects. Publications [1, 3, 6, 8] present methods associated with construction-related risk management. In article [1] a systematic approach to extending the Technique for Order Preference by Similarity to Ideal Solution has been proposed for a risk assessment model selection, in the context of a fuzzy decision-making environment. In article [3] a Risk Management Maturity System has been proposed. The system consists of three functional components: capabilities, assessment, and evolution. It is based on an integrated application of theories and tools drawing from experience in systems engineering, emergence and adaptation techniques, and multi-criteria decision making. The resulting appraisal of project risk is a multidimensional assessment corresponding to specific risk characteristics. In article [6] the author has described the preparation of Project Risk Assessment Methodology and its mitigation in complex construction projects. The author has developed a function which makes it possible to categorize any risks into one of the five categories. It is a combination of probability and the impact of one of these items: people and their safety or budget, cost, schedule and planning or quality and performance. The relationship between risks impacts and their level of likelihood has been also investigated. In paper [8] the author has proposed the method of identification and preliminary quantification of risk. Risk reduction techniques, the method of risk allocation in construction schedules, the contingency plan development, and the assessment of the effectiveness of risk analysis has also been presented. This approach enables verification of the solutions and development of risk-related relevance coefficients for assessing project cost and time. This particular method of risk identification works as well as cost and time of the project is comparable to the PERT method which has been presented in articles [4, 5]. The most important practical consequence of the author's research is that instead of the time-consuming and costly process of selecting the proper activity duration distributions, planners should devote more effort to adequately determine the activity duration times. On the basis of these publications one can conclude that there is still a knowledge gap between practical needs and available theoretical solutions. There is no universal method of project risk assessment which is commonly used. In particular, this applies to risk assessment of works on the construction sites. The method of construction risk assessment that is proposed in this paper is an attempt to address this gap. Based on the construction risk identification the schedules and cost estimates are developed. The risk of final deadline delay and the risk of total cost increase of construction completion are calculated. Risks are analyzed in the interval $[1, 0]$.

2. RULES OF CONSTRUCTION RISK ASSESSMENT

Construction risk should be evaluated according to the conditions of works execution and resource requirements. In the case of high changeable circumstances, favorable, moderate, and difficult conditions of construction should be considered. Requirements of object structure for such conditions are described by using the design characteristics model of structure S as follows:

$$(1) \quad S = \langle G, L \rangle$$

$G = \langle Y, U, P \rangle$ – coherent and a-cyclic unigraph with a single initial node and a single final node that describes interdependence and permissible sequences of works execution,

$Y = \{y_1, \dots, y_i, \dots, y_k, \dots, y_m\}$ – set of the nodes of the graph representing initial and final events for each work $u_j \in U$; the nodes mean: y_i initial and y_k final node of the work u_j execution;

$U = \{u_1, \dots, u_j, \dots, u_l, \dots, u_n\}$ – set of the arcs (arrows) of the graph representing relatively independent works (activities) that are constrained by initial $y_i \in Y$ and final $y_k \in Y$ nodes;

$P \subset Y \times U \times Y$, $\langle y_i, u_j, y_k \rangle \in P$ – three-term relation that assigns to each arc $u_j \in U$ the initial node $y_i \in Y$ and final node $y_k \in Y$;

$L: U \rightarrow R^+$ – function defined on the set U of arcs of the graph G that determines the certain l_j or expected $E[L_j]$ values of the variables L_j and describes the size of works $u_j \in U$.

Requirements of works execution are described by using the construction technology model \mathcal{L} :

$$(2) \quad \mathcal{L} = \{ \langle H, K, T \rangle, S \}$$

$H = \{H^1, H^2, \dots, H^r, \dots, H^s\}$ – set of rational or optimal teams H^r for works $u_j \in U^r$ execution,

$T: (H \times U) \rightarrow R^+$ – function defined on the set H of teams H^r which determines the expected values $E[T_{j,r}]$ of random variables $T_{j,r}$ and describes the expected duration of works $u_j \in U^r \subset U$.

$K: (H \times U) \rightarrow R^+$ – function defined on the set H of teams H^r which determines the expected values $E[K_{j,r}]$ of random variables $K_{j,r}$ and describes the expected cost of works $u_j \in U^r \subset U$.

3. SCHEDULING CONSTRUCTION

Based on the design characteristics model of structure $S = \langle G, L \rangle$ and the construction technology model $\mathcal{L} = \{ \langle H, K, T \rangle, S \}$, the schedules and cost estimates can be developed. In the risk conditions, the schedules determine the expected final deadlines $E[T]$, $E[T^p]$, $E[T^n]$ and the cost estimates determine the expected total costs $E[K]$, $E[K^p]$, $E[K^n]$ for the moderate, favorable, and difficult construction conditions, respectively. These values can be calculated using the suitable software package for works-scheduling and software package for cost estimation. In order to explain the problem, the schedules of works execution are developed as follows:

- calculate the expected earliest final deadline $E[V_m]$ of construction S completion by using resources \mathcal{L} , and the expected early start $E[V_i]$, that is calculate the values of the variables $\langle E[V_1], \dots, E[V_i], \dots, E[V_m] \rangle$ that minimize the function $E[T]$:

$$E[T] = \min \sum_{i=1}^{i=m} E[V_i],$$

- (3) under the constraints: $E[V_k] - E[V_i] \geq E[T_j]$ dla $u_j \in U, j = 1, 2, \dots, n, \langle y_i, u_j, y_k \rangle \in P$
 $E[V_i], E[V_k] \geq 0$ dla $i = 1, 2, \dots, m$

- calculate the expected earliest final deadline $E[V_m]$ of construction S completion by using resources \mathcal{L} , and the expected late start $E[V_i]$, that is calculate the values of the variables $\langle E[V_1], \dots, E[V_i], \dots, E[V_m] \rangle$ that maximize the function $E[T]$:

$$E[T] = \max \sum_{i=1}^{i=m} E[V_i],$$

- (4) under the constraints: $E[V_k] - E[V_i] \geq E[T_j]$ dla $u_j \in U, j = 1, 2, \dots, n, \langle y_i, u_j, y_k \rangle \in P$
 $E[V_m] \leq E[V_m]$
 $E[V_i], E[V_k] \geq 0$ dla $i, k = 1, 2, \dots, m$.

The above problems one can solve by using any standard computer program.

4. CONSTRUCTION RISK ASSESSMENT

Construction risk assessment here is strictly connected with the schedules and with the cost estimates of construction completion. Risk assessment always refers to the specific conditions of construction, and is calculated in accordance with the normal distribution of the random variables T , T^p , T^n , K , K^p and K^n . Taking this into consideration, the risk of final deadline delay and the risk of total cost increase are calculated. Based on the solutions of scheduling problems, the critical paths (U^{CP}) are determined. For works $u_j \in U^{CP}$, the variances of final deadline $D^2(T)$, $D^2(T^p)$, $D^2(T^n)$ are calculated. For works $u_j \in U$, the variances of total cost $D^2(K)$, $D^2(K^p)$, $D^2(K^n)$ are calculated. Based on the values $E[T]$, $E[T^p]$, $E[T^n]$, $E[T_{j,r}]$, $E[T_{j,r}^p]$, $E[T_{j,r}^n]$, $D^2(T)$, $D^2(T^p)$, $D^2(T^n)$ and $E[K]$, $E[K^p]$, $E[K^n]$, $D^2(K)$, $D^2(K^p)$, $D^2(K^n)$, $E[K_{j,r}]$, $E[K_{j,r}^p]$, $E[K_{j,r}^n]$ one can calculate the risk of final deadline delay of construction completion and the risk of total cost increase of construction completion. The risks of final deadline delay $p(t) = P(E[T] \geq t)$, $p^p(t) = P(E[T^p] \geq t)$ and $p^n(t) = P(E[T^n] \geq t)$ for the specified values of final deadline t and the normal distribution of the random variable T one can calculate as follows:

$$(5) \quad p(t) = P[E[T] \geq t] = 1 - P[E[T] \leq t] \approx 1 - Z \left[\frac{t - E[T]}{\sqrt{D^2[T]}} \right]$$

t – specified value of deadline of construction S completion which is probable and can be considered under the contract negotiations;

Z – standardized normal random variable.

The risks of the total cost increase $p(k) = P(E[K] \geq k)$, $p^p(k) = P(E[K^p] \geq k)$ and $p^n(k) = P(E[K^n] \geq k)$ for specified values of total cost k and normal distribution of the random variable K one can calculate as follows:

$$(6) \quad p(k) = P[E[K] \geq k] = 1 - P[E[K] \leq k] \approx 1 - Z \left[\frac{k - E[K]}{\sqrt{D^2[K]}} \right]$$

k – specified value of the cost of construction S completion, which is probable and can be considered under the contract negotiations.

Using the formulas (5) and (6) the risks $p(t)$, $p^p(t)$, $p^n(t)$, $p(k)$, $p^p(k)$ and $p^n(k)$ for successive values t and k can be presented on the charts of risk of final deadline delay, and the charts of risk of total cost increase of construction completion. These charts are referred to as contingency charts of construction delays and contingency charts of construction cost overruns. These charts characterize changes of the risk of final deadline delay $p(t)$ and changes of the risk of the total cost increase $p(k)$ in the interval $[1, 0]$ for consecutive values of the total costs $k \in [k_{\min}, k^{\max}]$ and the final deadlines $t \in [t_{\min}, t^{\max}]$. It means that: $p(k_{\min}) = 1$, $p(k_{\max}) = 0$, $p(t_{\min}) = 1$, $p(t_{\max}) = 0$. The different values t and k can be analyzed during the contract negotiations as the final deadline and total cost of construction completion. Taking into account the additional conditions of decision-making, the values that are acceptable in given situation should be settled in the contract.

5. RISK ASSESSMENT OF SMALL BRIDGE CONSTRUCTION

In order to explain this method, a small bridge erection is considered. The construction of the bridge is described in the design documentation and in the primary cost estimation. The design characteristics model of bridge structure S has been developed in accordance with formula (1). The bridge construction technology model \mathcal{L} has been developed in accordance with the formula (2). Detailed data for the construction of the bridge described by using these models has been tallied in Table 1. On the basis of this data, the schedules and cost estimates of the bridge construction in the moderate, favorable, and difficult conditions of construction have been developed. Here, all schedules have been made as solutions of the scheduling problems of bridge construction according to the equations (3) and (4) for the moderate, favorable, and hard construction conditions. For solving these problems MS Excel was used. Results of the calculations are tallied in Table 2.

The solutions of the problems determine ES – the expected early start $E[V_i]$, and LS – the expected late start $E[V_i]$ for each node $y_i \in Y$ for the works $U_j \in U$, $\langle y_i, u_j, y_k \rangle \in P$. The solutions determine also the expected final deadlines $E[T]$, $E[T^p]$ and $E[T^n]$ of bridge completion. For each solution expected total costs $E[K]$, $E[K^p]$ and $E[K^n]$ of bridge completion have been calculated. Then, using formula (5) and taking into account the work that belongs to the critical path, for subsequent values of t the risk of final deadline delay of construction completion has been calculated. Next, using formula (6) and taking into account all works of bridge construction, for subsequent values of k the risks of the total cost increase of construction completion have been calculated.

Table 1. Data for the design characteristics model of bridge structure S and bridge construction technology model L

No	Title	Duration of the works implementation (days)										Cost of the works implementation (PLN)						Dummy activities											
		Moderate conditions					Difficult conditions					Moderate conditions			Favorable conditions			Difficult conditions	Favorable conditions										
		$E[T]$	\bar{T}_j	\tilde{T}_j	\bar{T}_j	$E^p[T_j]$	$E^p[T_j]$	$E^p[T_j]$	$E^p[T_j]$	$E^p[T_j]$	$E^p[T_j]$	$E[K_j]$	\bar{K}_j	\tilde{K}_j	\bar{K}_j	$E^p[K_j]$	$E^p[K_j]$			$E^p[K_j]$	y_i	y_k							
1	Land development and preconstruction works	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22								
2	Temporary support for structural assembly and erection	1	1	2	$E[V_1]$	$E[V_2]$	1	44	34	43	56	43	45	58 560	46 080	57 600	74 880	57 792	60 432	2	3								
3	Abutment left	2	2	7	$E[V_2]$	$E[V_7]$	2	46	36	45	59	45	47	39 396	31 000	38 750	50 375	38 879	40 655	2	4								
4	Abutment right	3	3	5	$E[V_3]$	$E[V_5]$	3	167	131	164	213	165	172	1 683 600	1 324 800	1 656 000	2 152 800	1 661 520	1 737 420	5	7								
5	Embankment left	4	4	6	$E[V_4]$	$E[V_6]$	4	140	110	138	179	138	145	1 491 111	1 173 334	1 466 667	1 906 667	1 471 556	1 538 778	6	7								
6	Embankment right	5	5	8	$E[V_5]$	$E[V_8]$	5	56	44	55	72	55	58	575	453	566	736	568	594										
7	Bridge span	6	6	8	$E[V_6]$	$E[V_8]$	6	74	58	73	95	73	77	529	416	520	676	522	546										
8	Bituminous pavement	7	7	8	$E[V_7]$	$E[V_8]$	7	224	176	220	286	221	231	2 501 000	1 968 000	2 460 000	3 198 000	2 468 200	2 580 950										
9	Finishing works	8	8	9	$E[V_8]$	$E[V_9]$	8	13	10	13	17	13	14	20 333	16 000	20 000	26 000	20 067	20 983										
9	Finishing works	9	9	10	$E[V_9]$	$E[V_{10}]$	9	49	38	48	62	48	50	68 625	54 000	67 500	87 750	67 725	70 819										
Expected total cost of the construction completion												5 863 730												5 767 603		5 786 528		6 051 177	

Table 2. Results of the calculations

No	Title	u_j	y_i	y_k	$E[V_i]$	$E[V_k]$	H^r	Expected early start (ES) and expected late start (LS) Expected values $E[V_j]$ (days)												Expected cost $E[K_j]$ of the works execution (PLN)									
								Moderate conditions						Difficult conditions						Moderate conditions			Difficult conditions						
								ES	ES	ES	ES	ES	ES	ES	ES	ES	ES	ES	ES	ES	ES	ES	ES	ES	ES	ES			
1	Land development and preconstruction works	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17													
1	Land development and preconstruction works	1	1	2	$E[V_1]$	$E[V_2]$	1	0	0	0	0	0	0	0	58 560	57 792	60 432												
2	Temporary support for structural assembly and erection	2	2	7	$E[V_2]$	$E[V_7]$	2	44	44	43	43	45	45	45	39 396	38 879	40 655												
3	Abutment left	3	3	5	$E[V_3]$	$E[V_5]$	3	44	44	43	43	45	45	45	1 683 600	1 661 520	1 737 420												
4	Abutment right	4	4	6	$E[V_4]$	$E[V_6]$	4	44	44	43	43	45	45	45	1 491 111	1 471 556	1 538 778												
5	Embankment left	5	5	8	$E[V_5]$	$E[V_8]$	5	210	210	208	208	217	217	217	575	568	594												
6	Embankment right	6	6	8	$E[V_6]$	$E[V_8]$	6	184	210	182	208	190	190	190	529	522	546												
7	Bridge span	7	7	8	$E[V_7]$	$E[V_8]$	7	210	210	208	208	217	217	217	2 501 000	2 468 200	2 580 950												
8	Bituminous pavement	8	8	9	$E[V_8]$	$E[V_9]$	8	434	434	428	428	448	448	448	20 333	20 067	20 983												
9	Finishing works	9	9	10	$E[V_9]$	$E[V_{10}]$	9	447	447	441	441	441	462	462	68 625	67 725	70 819												
Expected final deadline of the construction project completion								496												512						5 863 730		6 051 177	

The risks have been shown on the risk charts of construction completion (Figs. 1 and 2).



Fig. 1. Risk charts of the final deadline delay of bridge construction completion

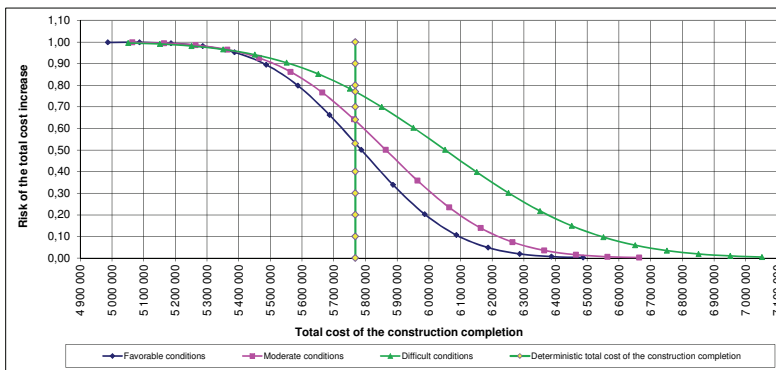


Fig. 2. Risk charts of the total cost increase of bridge construction completion

6. CONCLUSIONS

The method of construction risk assessment that has been presented in the paper allows contractors to examine changes of construction risk depending on conditions of construction and depending on different values of the final deadline t and the total cost k of bridge completion. All acceptable changes of these quantities in the interval $(1, 0]$ are shown in the risk charts. The smallest values t_{\min} and k_{\min} of the final deadlines and the total costs are calculated for the probabilities of exceeding $p(t)=1$ and $p(k)=1$. For these values, the final deadline and the total cost of construction

completion will most likely be exceeded. The greatest values t^{\max} and k^{\max} of the final deadlines and total costs are calculated for the probabilities of exceeding $p(t)=0$ and $p(k)=0$. For these values, the final deadline and the total cost of construction completion will most likely not be exceeded. Such values and the ranges of their changes have been calculated for the favorable, moderate, and difficult conditions of bridge completion. On the basis of such analysis one can evaluate the degree of threat that the fixed final deadline of the construction completion can exceed, and estimate the threat that the contracted total cost of construction completion can surpass. In Table 3 are shown the risks of final deadline delay $p(t)$, $p^p(t)$ and $p^n(t)$, and the risks of total cost increase $p(k)$, $p^p(k)$ and $p^n(k)$ of bridge completion for the final deadline t and the total cost k , equal to the deterministic values that are values which are calculated without risk analysis. Unfortunately these risks are unknown in the deterministic approach.

Table 3. The risk of final deadline delay and the risk of total cost increase of bridge construction completion for the deterministic values of the final deadline and total cost of construction

Conditions of bridge construction	Risk of final deadline delay of bridge completion for 488 days	Risk of total cost increase of bridge completion for 5 767 603 PLN
Favorable	0,53	0,53
Moderate	0,64	0,64
Difficult	0,77	0,77

Taking into account the results of calculations presented in the Table 3, it is important to clarify that the risks of the final deadline delay and the risks of the total cost increase of construction completion are equal because of the relatively short duration of the construction and the stable prices of labor and construction products during works execution. In such situations, this conclusion is also true for other permissible values of the final deadline and total cost of construction completion.

Finally, one can confirm that the method of construction risk assessment is theoretically ready to use. But, in practice, one should say that there are two basic conditions in order to use it. First, the method of identification of random events and their impact on execution and results of the works should be improved, for example by using an expert system. In particular this applies to the quantitative estimation of the impact of the disruptions on works execution. Secondly, it is necessary to develop software that would be convenient in to use in this situation. The software should be fully compatible with the cost estimation software. At present, these conditions are in the process of being analyzed.

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Fig. 2. Risk charts of total cost increase of bridge construction completion

Rys 2. Wykresy ryzyka wzrostu całkowitego kosztu zakończenia budowy mostu
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KWANTYTATYWNA OCENA RYZYKA BUDOWY

Słowa kluczowe: warunki ryzyka, harmonogramowanie, kosztorysowanie, ocena ryzyka, wykresy ryzyka

STRESZCZENIE

Realizacja robót budowlanych na placu budowy jest podstawowym etapem przedsięwzięcia budowlanego. W praktyce, z punktu widzenia realizacji robót na placu budowy lub przebudowy, montażu, remontu lub rozbiórki obiektu budowlanego, ryzyko czasu i kosztów często powinno być dokładnie badane ze względu na wpływ zdarzeń losowych na przebieg i wyniki wykonania robót. Jest wiele publikacji na temat metod identyfikacji i oceny ryzyka. Jednak opublikowane metody nie rozwiązują problemów oceny ryzyka, które generowane jest przez proces realizacji robót na placu budowy i jego otoczenie. Proponowana metoda jest koncepcją ilościowej oceny ryzyka robót budowlanych na placu budowy. Jest to końcowy etap analizy ryzyka. Poprzedza go identyfikacja uwarunkowań ryzyka, które zależą od losowej technologicznej charakterystyki konstrukcji, losowej charakterystyki dostępnych zasobów oraz losowych warunków realizacji robót. W pierwszym kroku analizy ryzyka są identyfikowane charakterystyki losowe oraz obliczane są oczekiwany czas i oczekiwane koszty poszczególnych robót budowlanych. W drugim kroku, wyznaczane są harmonogramy i kosztorysy robót dla przewidywanych warunków losowych budowy. W trzecim kroku, badane i oceniane jest ryzyko opóźnienia ostatecznego terminu zakończenia budowy $p(t_m)$ i ryzyko przekroczenia całkowitych kosztów budowy $p(k_m)$. Ryzyka te badane są w przedziale $[1, 0]$. Ryzyka $p(t) = 1$ i $p(k) = 1$ wyznaczają wartości końcowe czasu t_m i kosztów k_m ukończenia budowy, które najprawdopodobniej będą przekroczone. Zatem wartości równe i mniejsze od t_m i k_m są niedopuszczalne i nie powinny być ustalane, jako wartości umowne. Ryzyka $p(t) = 0$ i $p(k) = 0$ wyznaczają wartości końcowe czasu t_m i kosztów k_m ukończenia budowy, które najprawdopodobniej nie będą przekroczone i formalnie mogą być ustalane, jako wartości umowne. Oczywiście, wszystkie większe wartości końcowe również nie będą przekroczone i także mogą być wyznaczone, jako umowne wartości czasu i kosztów zakończenia budowy. W przypadku silnie zmiennych warunków realizacji identyfikację i ocenę ryzyka proponuje się przeprowadzić dla pomyślnych, przeciętnych i trudnych warunków budowy. W każdym rozpatrywanym przypadku ryzyko analizowane powinno być w zależności od losowych wymagań technologicznych konstrukcji obiektu oraz technologiczno-organizacyjnych losowych uwarunkowań zasobów, które mogą być wykorzystane do wykonania robót. W pracy, wymagania technologiczne konstrukcji obiektu są opisywane za pomocą modelu technologii konstrukcji obiektu $S = \langle G, L \rangle$. W modelu tym graf G opisuje zbiór i współzależność technologiczną przedmiarowanych robót. Natomiast funkcja L opisuje rozmiar przedmiarowanych robót. Wielkości te określane są na podstawie analizy projektu budowlanego. Wymagania technologiczne, organizacyjne i zasobowe robót są opisywane za pomocą modelu technologii budowy $\mathcal{L} = \{ \langle H, K, T \rangle, S \}$. Zbiór H zawiera racjonalne lub optymalne brygady robocze, które zorganizowane (zaprojektowane) zostały do wykonania robót S . Wartości funkcji K i T opisują odpowiednio koszty i czas realizacji robót S przez brygady robocze zbioru H . Wartości tych funkcji są określane na podstawie analizy modelu S , dyspozycyjnych zasobów i warunków wykonania robót. Wykorzystując dane opisane za pomocą tak zdefiniowanych modeli są opracowywane harmonogramy i kosztorysy robót w warunkach probabilistycznych przeciętnych, pomyślnych i trudnych. Mogą one być opracowywane za pomocą typowych programów do harmonogramowania i kosztorysowania robót. W pracy, w celu przygotowania danych do bezpośredniej oceny ryzyka, formułowane i rozwiązywane są problemy harmonogramowania robót dla modeli S i \mathcal{L} , jako zagad-

nienia programowania liniowego. W ten sposób wyznaczane są ostateczne oczekiwane końcowe terminy $E[T]$, $E[T^p]$, $E[T^n]$ oraz oczekiwane całkowite koszty $E[K]$, $E[K^p]$, $E[K^n]$ wykonania robót w warunkach przeciętnych, pomyślnych i trudnych. Zakładając w tych warunkach rozkład normalny losowych zmiennych czasu trwania T , T^p , T^n oraz całkowitych kosztów K , K^p i K^n budowy, dla poszczególnych harmonogramów są wyznaczane wariancje ostatecznych terminów $D^2(T)$, $D^2(T^p)$, $D^2(T^n)$ i wariancje całkowitych kosztów $D^2(K)$, $D^2(K^p)$, $D^2(K^n)$ zakończenia budowy. Na podstawie tych wartości obliczane jest ryzyko czasu i kosztów budowy. Dla ciągu dopuszczalnych wartości t terminu końcowego wykonania robót jest obliczane ryzyko opóźnienia ostatecznego terminu zakończenia budowy za pomocą zależności: $p(t) = P(E[T] \geq t)$ w warunkach przeciętnych, $p^p(t) = P(E[T^p] \geq t)$ w warunkach pomyślnych i $p^n(t) = P(E[T^n] \geq t)$ w warunkach trudnych. Dla ciągu wartości k całkowitych kosztów wykonania robót jest obliczane ryzyko przekroczenia całkowitych kosztów budowy za pomocą zależności: $p(k) = P(E[K] \geq k)$ w warunkach przeciętnych, $p^p(k) = P(E[K^p] \geq k)$ w warunkach pomyślnych i $p^n(k) = P(E[K^n] \geq k)$ w warunkach trudnych. Następnie dla ciągu wartości końcowych t i k , wartości $E[T]$, $E[T^p]$, $E[T^n]$, $E[K_{j,r}]$, $E[T_{j,r}^p]$, $E[T_{j,r}^n]$ oraz $D^2(K)$, $D^2(K^p)$, $D^2(K^n)$, $E[K_{j,r}]$, $E[K_{j,r}^p]$ i $E[K_{j,r}^n]$ są opracowywane wykresy ryzyka opóźnienia ostatecznego terminu $p(t)$, $p^p(t)$ i $p^n(t)$ oraz wykresy ryzyka wzrostu całkowitych kosztów $p(k)$, $p^p(k)$ i $p^n(k)$ zakończenia budowy. Jak wynika z przedstawionych zależności, miarą ryzyka jest prawdopodobieństwo opóźnienia ostatecznego terminu zakończenia budowy $p(t)$ i prawdopodobieństwo wzrostu całkowitych kosztów $p(k)$ ukończenia budowy dla określonych wartości końcowego terminu t i całkowitych kosztów k . W celu określenia granicznych wartości czasu i kosztów ukończenia budowy wyznaczane są przedziały dopuszczalnej (realizowalnej) zmienności końcowego terminu $t \in [t_{\min}, t_{\max}]$ i całkowitych kosztów $k \in [k_{\min}, k_{\max}]$ ukończenia budowy. Wartości t_{\min} i k_{\min} , dla których $p(t_{\min}) = 1$ i $p(k_{\min}) = 1$, wyznaczają najkrótszy czas i najmniejsze koszty budowy, które będą najprawdopodobniej przekroczone. Wartości t_{\max} i k_{\max} , dla których $p(t_{\max}) = 0$ i $p(k_{\max}) = 0$, wyznaczają limity czasu i kosztów budowy, które najprawdopodobniej będą nieprzekroczone. Wszystkie wartości t i k zawarte w przedziałach $(t_{\min}, t_{\max}]$ oraz $(k_{\min}, k_{\max}]$ są dopuszczalne (realizowalne). Jednak ryzyko związane z ich dotrzymaniem może się znacznie różnić. Oczywiście wszystkie wartości t_k i k_k zakończenia budowy większe od t_{\max} i k_{\max} mogą być również przyjęte, jako umowne wartości dyrektywne zakończenia budowy, gdyż ryzyko ich przekroczenia wynosi zero, czyli: $p(t > t_{\max}) = 0$ i $p(k > k_{\max}) = 0$. Znajomość limitów dolnych t_{\min} i k_{\min} , górnych t_{\max} i k_{\max} oraz wykresów ryzyka umożliwia wykonawcy i inwestorowi ustalenie wartości kontraktowych końcowego terminu i całkowitych kosztów zakończenia budowy dla określonego poziomu ryzyka.