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Analysis of the Accuracy of Uncertainty Noise Measurement

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The paper formulates some objections to the methods of evaluation of uncertainty in noise measurement which are presented in two standards: ISO 9612 (2009) and DIN 45641 (1990). In particular, it focuses on approximation of an equivalent sound level by a function which depends on the arithmetic average of sound levels. Depending on the nature of a random sample the exact value of the equivalent sound level may be significantly different from an approximate one, which might lead to erroneous estimation of the uncertainty of noise indicators. The article presents an analysis of this problem and the adequacy of the solution depending on the type of a random sample.

Keywords: acoustics, noise measurement, uncertainty in measurement, probability distribution

1. Introduction

There exist standards for determining uncertainty in noise measurement, among others: ISO 9612 (2009) and DIN 45641 (1990). They are based on the rules of uncertainty estimation developed by seven international metrology organizations and described in the "Guide to the Expression of Uncertainty Measurement" (International Organization for Standardization, 1995). However, application of the methods described in the above standards may lead to some nonverisimilar results due to adopted therein assumptions. The most serious objections relate to the assumption that the random variable $L_{A,i}$, $i = 1, 2, \ldots, n$ which describes the sample of the measured sound level, has a normal probability distribution. In practice, this may be not achieved especially for samples obtained from road or train noise measurements, which rarely have a normal distribution (Wszołek, Kłaczyński, 2006; Batko, Przysucha, 2013; Gałuszka, 2010; Batko, Bal, 2014). Moreover, the nature of the sample's probability distribution often remains unverified.

Therefore, the following questions arise: how high is the probability of a mistake by applying this method for any type of noise and can such an error may be negligible in the uncertainty determination? If it cannot, calculation of the noise indicators and their uncertainties using the methods specified in the discussed standards may be incorrect. As a consequence, some wrong environmental decisions might be taken.

2. Uncertainty in noise measurement by ISO 9612 (2009) and DIN 45641 (1990)

The standards ISO 9612 (2009) and DIN 45641 (1990) describe the procedure for determining the uncertainty in noise measurement. It is based on an approximation of noise perception described by logarithmic average sound levels (Thiery, Ognedal, 2008):

$$L_{Aeq} = 10 \log \left(\frac{1}{n} \sum_{i=1}^{n} 10^{0.1 L_{A,i}} \right) \tag{1}$$

by the arithmetic average of random samples from noise measurements:

$$\overline{L} = \frac{1}{n}\sum_{i=1}^{n} L_{A,i}. \tag{2}$$

This relation is given by the equation:

$$L_{Aeq} = 10\log\left(\frac{1}{n}\sum_{i=1}^{n} 10^{0.1L_{A,i}}\right) \approx \overline{L} + 0.115s^2, \tag{3}$$

where

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(L_{A,i}-\overline{L})^2}. \tag{4}$$

Basing on Eqs. (3) and (4), the expanded uncertainty of the results obtained from noise measurement is estimated:

$$U = \pm\sqrt{\frac{s^2}{n} + \frac{0.026s^4}{n-1}}\,t_{\alpha;n-1}, \tag{5}$$

specified by the coverage factor $t_{\alpha;n-1}$ which is the t-Student quintile for the assumed required confidence level equal to $1-\alpha$, where α is the significance level (usually $\alpha = 0.05$). Finally, the uncertainty of measurement results can be described by the defined above interval of uncertainty:

$$\widehat{L}_{Aeq} \mp U. \tag{6}$$

The authors decided to verify the reliability of the described method by applying it to assess the uncertainty of some sound level measurements. The study was based on the results of sound level measurements collected at a station of continuous noise monitoring in Cracow. For random sample obtained from the measurements ($n = 199$), the exact value of the equivalent sound level L_{Aeq}, and the approximate value $\widehat{L}_{Aeq}$ were determined with the uncertainty calculated from formula (5). All the results are presented in Table 1.

Table 1. Exact and approximate values of the equivalent sound level obtained from the exemplary measurements with a calculated interval of uncertainty (ISO 9612 (2009), DIN 45641 (1990)).

L_{Aeq}	$\widehat{L}_{Aeq}$	$U(95\%)$
72.2 dB	74.1 dB	0.8 dB

The real value of the logarithmic average is equal to 72.2 dB and it does not fall within the 95% confidence interval. Moreover, the difference between these values (equal to 1.9 dB) is almost three times bigger than half of the U interval‘s length. Therefore, it is possible to conclude that using the relationship between the arithmetic and logarithmic means described by Eq. (3) the uncertainty in noise measurement might be improperly determined. The interval described by Eq. (6) may not cover the equivalent sound level calculated from the empirical data. Furthermore, the difference between the approximate and exact values can be even several times higher than the value of the expanded uncertainty U.

The authors suggest that such a discrepancy in determining the uncertainty in noise measurement may result from the nature of probability distribution of the random variable which describes the sample of the measured sound level. The used algorithm assumes that this random variable has a normal probability distribution, while this rarely happens in the case of road noise measurements (it could also concern other types of noise). Therefore, a detailed analysis of the accuracy of the approximate formula (3) was carried out. The authors tried to define how the probability distribution of the test's results influences this estimation. The study was based on numerical experiments, thus, it was possible to model various probability distributions.

3. Analysis of errors in an equivalent sound level approximation

The first step of the study was to analyse the error in an equivalent sound level approximation given by the formula (3). Assuming a normal random variable of sound levels, the derivation of this equation was presented by Thiery and Ognedal (2008). However, in the general case, it might be derived on the basis of the Delta method (Magiera, 2005) as shown below.

Let $Y = g(X)$ be a function having a derivative of all orders (that meets the Taylor's theorem), and let X be a random variable having moments of all orders. The essence of the Delta method is to expand the function $g(X)$ around the point $\mu = E(X)$, where $E(X)$ is the expected value of the random variable X. As a result, the following equation is obtained:

$$Y = g(X) = g(\mu) + (X-\mu)g'(\mu) + (X-\mu)^2\frac{g''(\mu)}{2!} + \ldots . \tag{7}$$

Taking the expected value of both sides of the expansion and using the equation:

$$E(X-\mu)g'(\mu) = 0 \tag{8}$$

we obtain:

$$E(Y) = g(\mu) + \frac{g''(\mu)}{2!}\sigma^2 + \varepsilon, \tag{9}$$

where

$$\sigma^2 = E(X-\mu)^2 \tag{10}$$

and

$$\varepsilon = \sum_{j=2}^{+\infty}\frac{g^{(j)}(\mu)}{j!}E(X-\mu)^j. \tag{11}$$

Omitting the expression (11) in the formula (9) we get the following approximation to the expected value of a random variable function:

$$E(Y) \approx g(\mu) + \frac{g''(\mu)}{2!}\sigma^2. \tag{12}$$

In the case of Eq. (3), which describes the approximation of the sound level's logarithmic mean by the arithmetic mean, the Delta method was used for the following function:

$$Y = 10^{0.1X}. \tag{13}$$

Expanding the function (13) in a Taylor series to the second order expression around the point $\mu = E(X)$, we obtain:

$$\begin{aligned} Y &\approx 10^{0.1\mu} + (X-\mu)10^{0.1\mu}\frac{\ln 10}{10} \\ &\quad + (X-\mu)^2\frac{10^{0.1\mu}}{2}\left(\frac{\ln 10}{10}\right)^2. \end{aligned} \tag{14}$$

Then for both sides of the equation we calculate the expected value:

$$\begin{aligned} E(Y) &\approx 10^{0.1\mu} + \sigma^2\frac{10^{0.1\mu}}{2}\left(\frac{\ln 10}{10}\right)^2 \\ &= 10^{0.1\mu}\left(1 + \frac{\sigma^2 \ln^2 10}{2\cdot 10^2}\right). \end{aligned} \tag{15}$$

Next, we count the decimal logarithm and multiply it by 10:

$$10\log E(Y) \approx \mu + 10\log\left(1 + \frac{\sigma^2 \ln^2 10}{2\cdot 10^2}\right). \tag{16}$$

The next step is this estimation is to approximate the logarithmic function by a linear function:

$$\ln(1+x) \approx x. \tag{17}$$

Using the formulas (15), (16), and (17), we finally obtain:

$$10\log E(Y) \approx \mu + \frac{\sigma^2 \ln 10}{20} = \mu + 0.115\sigma^2. \tag{18}$$

In the case of the assumption that the random variables X and Y have normal distributions, or in the case of an appropriately large measurement sample, from the Law of large numbers (Magiera, 2005), we can assume that:

$$E(Y) = \overline{Y} = \frac{1}{n}\sum_{i=1}^{n} 10^{0.1L_{A,i}} \tag{19}$$

and

$$E(X) = \overline{L} = \sum_{i=1}^{n} L_{A,i}. \tag{20}$$

Inserting Eqs. (19) and (20) into Eq. (18) we finally get the approximation of the equivalent sound level $\widehat{L}_{Aeq}$ given by the formula (3).

The analysis shows that this approximation should not generate large errors only if the values of the factor ε and variance σ^2 are small and for sufficiently large samples having the probability distribution close to a normal distribution. Therefore, all approximation errors have the following sources:

a) omission of words of higher orders in the Taylor series;

b) inaccurate approximation of the logarithm by a linear function;

c) in the case of small samples: lack of conformity of the expected value to the arithmetic mean.

Referring to point a) there was made an expansion of the function (13) in a Taylor series to the fourth word, where the skewness ρ and kurtosis K of the probability distribution appear:

$$\begin{aligned} 10\log E(Y) &\approx \mu \\ &\quad +10\log\left(1+\frac{\sigma^2\ln^2 10}{2!\cdot 10^2} - \rho\frac{\sigma^3\ln^3 10}{3!\cdot 10^3} + K\frac{\sigma^4 \ln^4 10}{4!\cdot 10^4}\right) \\ &= \mu + 10\log(1 + 0.0266\sigma^2 - \rho\cdot 0.002\cdot\sigma^3 \\ &\quad + K\cdot 0.0001\sigma^4). \end{aligned} \tag{21}$$

In the case of a normal distribution the skewness and kurtosis take the values of 0 and 3, respectively. Therefore, they do not have a significant impact on the accuracy of the approximation (3). Nevertheless, for some distributions characterized by a different nature, the omission of the third and fourth word in the Taylor expansion can generate approximation errors.

As mentioned in point b), inaccurate approximation of a logarithm by a linear function may have a significant impact on the approximation errors. The corresponding graph illustrating this statement is presented in Fig. 1.

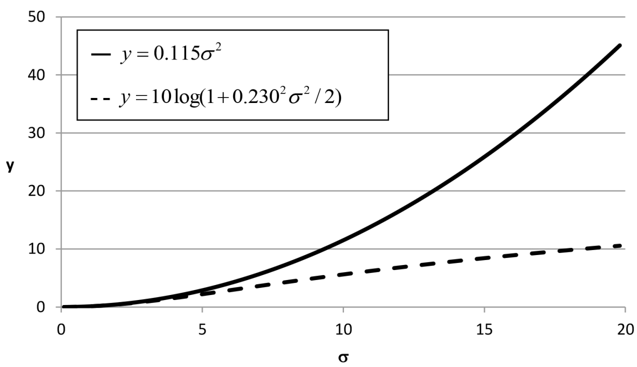

Fig. 1. Graph of a logarithmic function and its approximation given by the formula (17).

4. Analysis of the equivalent sound level approximation using numerical simulations

The next step of the research was to analyse the equivalent sound level approximation using some numerical simulations. The aim of the study was to determine how the nature of the probability distribution of the random variable (i.e., equivalent sound level) affects the value of the approximation error.

In order to make a simulation, 1000-element samples characterized by probability distributions being combinations of some normal distributions were generated. Such a choice makes it possible to account for any probability distribution, including right-skewed or left-skewed distributions and distributions with large or small kurtosis. These distributions may characterize any noise with different probabilistic structure derived from several interacting sources of noise. A combination of normal distributions was constructed as follows. Let us assume that there are two random variables:

$$X_1 \sim N(\mu_1, \sigma_1) \qquad \text{and} \qquad X_2 \sim N(\mu_2 \sigma_2) \tag{22}$$

of density functions, respectively:

$$f_1(x) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right)$$
$$\text{and} \qquad f_2(x) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right). \tag{23}$$

Then X is a random variable with the distribution described by a function f which is a combination of normal distributions f_1 and f_2:

$$f(x) = p f_1(x) + (1-p) f_2(x), \tag{24}$$

where $p \in [0,1]$ defines the share of the variable X_1 in the combination.

The results of the simulations were divided into three groups. The first one consists of the samples characterized by the distributions that are a combination of two distributions with equal variances and various modes:

1a) $X_1 \sim N(70,\sigma)$, $X_2 \sim N(80,\sigma)$, where $\sigma \in \{0.5, 1, \ldots, 20\}$ for $p \in \{0.1, 0.2, \ldots, 1\}$;

1b) $X_1 \sim N(70,\sigma)$, $X_2 \sim N(75,\sigma)$, where $\sigma \in \{0.5, 1, \ldots, 20\}$ for $p \in \{0.1, 0.2, \ldots, 1\}$;

1c) $X_1 \sim N(72,\sigma)$, $X_2 \sim N(75,\sigma)$, where $\sigma \in \{0.5, 1, \ldots, 20\}$ for $p \in \{0.1, 0.2, \ldots, 1\}$.

Applying different probability weights p one may determine the contribution of each distribution to the combination. For example, for p greater than 0.5 a larger share of the distribution of a higher mode is received while for p lower than 0.5 a distribution of a lower mode is prevalent Moreover, the change in the value of the parameter σ has also a significant impact on the characteristics of the probability distribution Hence, for the data 1a) with a decreasing σ the obtained distributions are characterized by a more pronounced bimodality. In the case of the combination 1b) the bimodality of the distribution disappears, whereas for the data 1c) the distribution becomes distinctly unimodal. On the other hand, the value of the parameter p determines the height of the probability distribution's bell for the different modes.

In Fig. 2 the values of the difference r between the exact equivalent sound level and an approximation (3) for the 1a), 1b), and 1c) data sets according to the value of the standard deviation s are presented.

The differences r which are shown in the graphs depend on the probability distribution of the random sample which is the basis for the calculation of L_{Aeq} and $\widehat{L}_{Aeq}$. Analysing Fig. 2(1c) it can be seen that the change of percentage of each distribution in the combination does not affect the value of the difference r. This is mainly due to the symmetry of the obtained distributions and their unimodality. A similar dependence is also shown in the graph 2(1b), as the distributions have similar properties. On the other hand, a significant impact of the distribution's nature on the difference r can be observed in Fig. 2(1a). For combinations with a smaller percentage of the distribution X_1 higher values of the difference r are obtained. The opposite situation occurs when the component X_1 dominates. This can be also observed for the exemplary data shown in Table 2. For $p = 0.3$ the inverted distribution relative to the distribution with $p = 0.7$ (the same kurtosis, the opposite skewness coefficient) is obtained. Nevertheless, the value of the

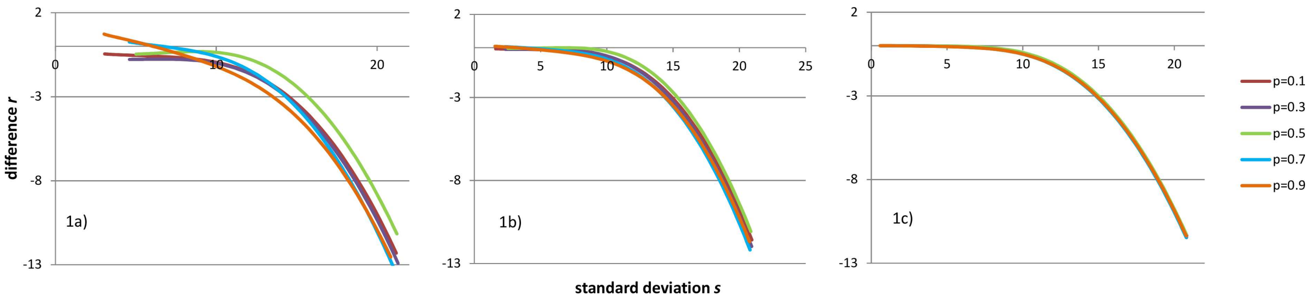

Fig. 2. Values of the difference r of the approximation (3) for the 1a), 1b), and 1c) data sets according to the value of the standard deviation s.

Table 2. Selected characteristics for the data 1a) which determine the probability distribution of the tested samples depending on the value of the parameter p.

μ_1 [dB]	σ_1 [dB]	μ_2 [dB]	σ_2 [dB]	p	$K-3$	ρ	s [dB]	$E(X)$ [dB]	L_{Aeq} [dB]	$\widehat{L}_{Aeq}$ [dB]	r [dB]
70	10	80	10	0.3	0.13	−0.09	11.37	76.59	90.09	91.45	−1.36
70	5	80	5	0.3	−0.15	−0.26	6.93	76.80	81.56	82.33	−0.77
70	0.5	80	0.5	0.3	−1.20	−0.86	4.61	76.99	78.65	79.43	−0.78
70	10	80	10	0.5	0.27	−0.07	11.39	74.59	88.93	89.51	−0.58
70	5	80	5	0.5	−0.21	−0.05	7.09	74.80	80.22	80.59	−0.37
70	0.5	80	0.5	0.5	−1.96	0.00	5.01	74.99	77.41	77.87	−0.47
70	10	80	10	0.7	0.16	−0.12	11.14	72.59	85.88	86.87	−0.99
70	5	80	5	0.7	−0.16	0.08	6.75	72.80	78.03	78.04	−0.02
70	0.5	80	0.5	0.7	−1.22	0.85	4.59	72.99	75.67	75.41	0.26

difference r is larger as to the absolute value in the case of the distribution with $p = 0.3$ Moreover, it is worth paying attention to the distribution's symmetry and the sign of the difference r. For distributions characterized by small values of σ and $p < 0.5$ the difference r is negative while for distributions with $p > 0.5$ this difference is positive.

The next group consists of samples characterized by the distributions that are a combination of normal distributions with different variances and a larger variance for the distribution of a higher mode:

2a) $X_1 \sim N(70, \sigma)$, $X_2 \sim N(80, \sigma + 5)$, where $\sigma \in \{0.5, 1, \ldots, 15\}$ for $p \in \{0, 0.1, \ldots, 1\}$;

2b) $X_1 \sim N(70, \sigma)$, $X_2 \sim N(75, \sigma + 5)$, where $\sigma \in \{0.5, 1, \ldots, 15\}$ for $p \in \{0, 0.1, \ldots, 1\}$;

2c) $X_1 \sim N(72, \sigma)$, $X_2 \sim N(73, \sigma + 5)$, where $\sigma \in \{0.5, 1, \ldots, 15\}$ for $p \in \{0, 0.1, \ldots, 1\}$.

Figure 3 shows the values of the difference r between the exact equivalent sound level and an approximation (3) for the 2a), 2b), and 2c) data sets according to the value of the standard deviation s.

If two distributions of distant modes are composed it is observed that the characteristics of the difference r are varied, even in the case of samples with small standard deviations. Moreover, large positive values of the differences are obtained in the case of a composition in which a larger part has the distribution of a higher mode and higher standard deviation (the distribution has a slight disorder on the left "tail"). This is confirmed by the data in Table 3.

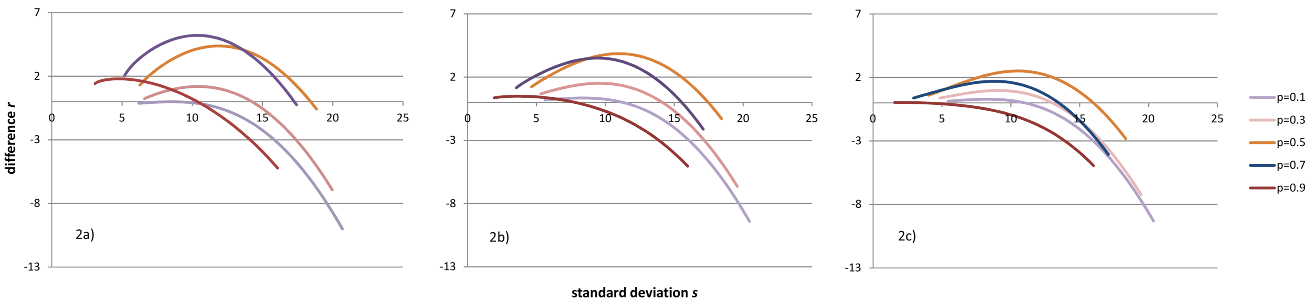

Fig. 3. Values of the difference r of approximation (3) for the 2a), 2b), 2c) data sets according to the value of the standard deviation s.

Table 3. Selected characteristics for the data 2a) which determine the probability distribution of the tested samples depending on the value of the parameter p.

μ_1 [dB]	σ_1 [dB]	μ_2 [dB]	σ_2 [dB]	p	$K-3$	ρ	s [dB]	$E(X)$ [dB]	L_{Aeq} [dB]	$\widehat{L}_{Aeq}$ [dB]	r [dB]
70	10	75	15	0.3	0.42	0.09	14.47	72.94	96.78	97.03	−0.25
70	5	75	10	0.3	0.57	0.24	9.47	73.15	84.98	83.47	1.51
70	0.5	75	5.5	0.3	0.45	0.57	5.31	73.34	77.27	76.59	0.68
70	10	75	15	0.5	0.68	0.17	13.36	71.90	95.80	92.43	3.36
70	5	75	10	0.5	1.25	0.46	8.46	72.11	83.71	80.34	3.37
70	0.5	75	5.5	0.5	1.91	1.17	4.64	72.30	76.01	74.77	1.24
70	10	75	15	0.7	0.55	0.07	12.03	70.90	90.45	87.54	2.91
70	5	75	10	0.7	1.55	0.42	7.13	71.11	80.05	76.95	3.10
70	0.5	75	5.5	0.7	5.18	1.89	3.54	71.30	73.90	72.74	1.16

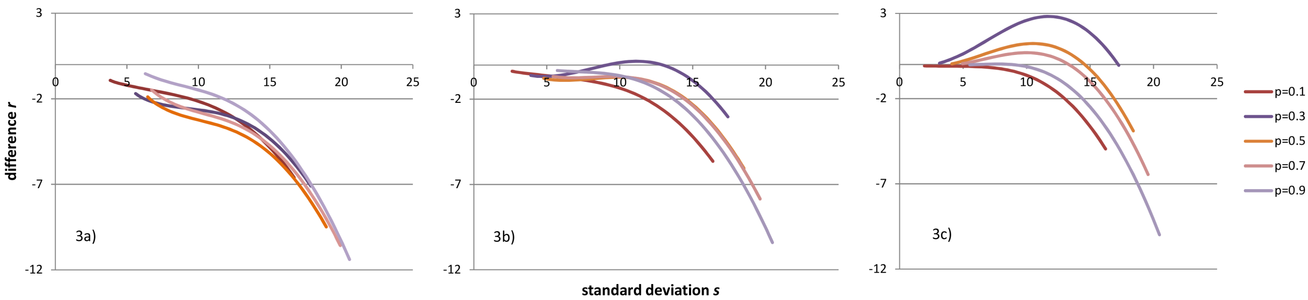

Fig. 4. Values of the difference r of approximation (3) for the 3a), 3b), 3c) data sets according to the value of the standard deviation s.

The third group consists of samples with distributions that are a combination of normal distributions with different variances and a lower variance for the distribution of a higher mode:

3a) $X_1 \sim N(70, \sigma+5)$, $X_2 \sim N(80, \sigma)$, where $\sigma \in \{0.5, 1, \ldots, 15\}$ for $p \in \{0, 0.1, \ldots, 1\}$;

3b) $X_1 \sim N(70, \sigma+5)$, $X_2 \sim N(75, \sigma)$, where $\sigma \in \{0.5, 1, \ldots, 15\}$ for $p \in \{0, 0.1, \ldots, 1\}$;

3c) $X_1 \sim N(72, \sigma+5)$, $X_2 \sim N(73, \sigma)$, where $\sigma \in \{0.5, 1, \ldots, 15\}$ for $p \in \{0, 0.1, \ldots, 1\}$.

In Fig. 4 the values of the difference r between the exact equivalent sound level and an approximation (3) for the 3a), 3b), and 3c) data sets according to the value of the standard deviation s are presented. In the case of the probability distributions with a smaller standard deviation for larger values of the mode (distributions with a disorder on the right "tail") the difference r fairly quickly converges to zero, with the exception of the data 3a), where there are significant differences.

Analysing all carried out simulations it can be stated that some sets of samples behave similarly. In the case of the probability distributions with a disorder on the left "tail" there is a noticeable dispersion of the values of the difference r (e.g. Fig. 3). The difference r between the exact equivalent sound level and an approximation (3) becomes smaller if the probability distributions with a disorder on the right "tail" are considered (e.g., Fig. 4: 3b) and 3c)). Finally, in the case of the symmetrical probability distributions the discussed differences quickly converge to zero and behave stably regardless of changes in the value of p (e.g. Fig. 2: 1b) and 1c)).

5. Summary and conclusions

The article formulates some objections to the methods of evaluation of uncertainty in noise measurement which are presented in two standards: ISO 9612 (2009) and DIN 45641 (1990). The proposed solutions refer to the rules of uncertainty estimation developed by seven international metrology organizations and described in the "Guide to the Expression of Uncertainty Measurement" (International Organization for Standardization, 1995). The adaptation of these rules for the estimation of uncertainty in noise measurement may lead to some non-verisimilar results due to the assumptions made in this method.

In particular, the article analyses the behaviour of the average logarithmic approximation of the arithmetic mean, which is used in determining the uncertainty in noise measurement. The analyses indicate the possible use of the approximation (3) only in the case of samples with symmetric unimodal distributions with the kurtosis and skewness coefficient close to zero and a small standard deviation. For samples with distributions characterized by disturbances and slight bimodality the approximation might produce large errors, even for small standard deviations. It happens especially in the case of the probability distribution of traffic noise measurement's samples: they are left-skewed and with disturbances in the left "tail" (Gałuszka, 2010; Wszołek, Kłaczyński, 2006; Przysucha, 2013). In this situation, the value of the combination's component which causes a disorder does not significantly affect the increase in the standard deviation and the value of the logarithmic mean. Nevertheless, it influences the shift of the mean value. Then, it may be impossible to calculate the uncertainty of L_{Aeq} from the approximation (3), since the obtained uncertainty range will not match the true value with a certain probability.

The presented verification of some commonly used methods to determine the uncertainty in the noise measurements (ISO 9612 (2009) and DIN 45641 (1990)) induces to search for and apply other solutions based for example on some measuring verified methods, i.e. the bootstrap method (Batko, Stępień, 2010; 2014), reduction interval arithmetic (Batko, Pawlik, 2012a; 2012b), time series analysis (Batko, Bal, 2010), or the propagation of probability distributions (Batko, Przysucha, 2010; 2014). Finally, it should be added that not only sound noise measurements assume a normal probability distribution of results. This assumption, often incorrect, is commonly used in other acoustic measurements, e.g., of acoustic structures' parameters (Kamisiński *et al.*, 2012; Szeląg *et al.*, 2013). Therefore, it is important to choose carefully the method for determining the acoustic measurements uncertainty due to the specific probability distribution of the results.

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