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CPC-based currents' description in three-phase four-wire systems with asymmetrical and nonsinusoidal waveforms

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Abstract. This article aims to explain the capability of describing three-phase systems powered from the source with asymmetrical nonsinusoidal voltage waveforms. Indicating the physical components of the currents associated closely with specific physical phenomena facilitates separating the unbalanced components from the active and reactive ones. All those currents, excluding the active current and the scattered current, contribute to the existing unbalanced and reactive power. This article presents the decomposition of currents based on the Currents' Physical Components Theory. When decomposing currents, it is assumed that the load is linear and unchanging in time, and the voltage supply is asymmetrical and nonsinusoidal.

Key words: Currents' Physical Components (CPC), power theory, asymmetric voltage, nonsinusoidal waveforms.

1. Introduction

Transmission of electric energy from sources to loads through power systems, where it is transformed adequately to the needs of the consumer, is described by power theories [1–6]. Over 100 years of energy transmission, numerous different approaches have been created.

Power theory describes the power properties of electrical systems in a frequency domain or in a time domain. The characterization in the time domain, because of the speed of calculation, is primarily used to control semiconductor devices in the active or hybrid power filters [7–12]. The most common time-domain methods are [8, 11–13]. The description in the frequency domain [1, 3], using Fourier Transform, causes delays in the measuring length. However, the methods based on frequency description are more precise and are also used to generate the reference current of the active power filter [7–9].

In the aforementioned power theories, the mathematical description and, consequently, the obtained results are correct on the assumption that the voltage source is symmetrical.

In publications [14–18] a description of asymmetrical sinusoidal three-phase, three-wire, or four-wire power systems is presented. Also, in [14, 15, 19–21], the possibility of building a balancing and reactive power compensator has been demonstrated.

This publication proposes an enlargement of the Currents' Physical Components (CPC) Theory for asymmetrical nonsinusoidal three-phase four-wire systems, i.e. circuits with a neutral conductor (N).

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2. Currents' Physical Components (CPC) Theory in three-phase four-wire systems at nonsinusoidal and asymmetric voltage waveforms

An unbalanced linear time-invariant load (LTI) powered from a source with nonsinusoidal and asymmetrical waveforms is shown in Fig. 1.

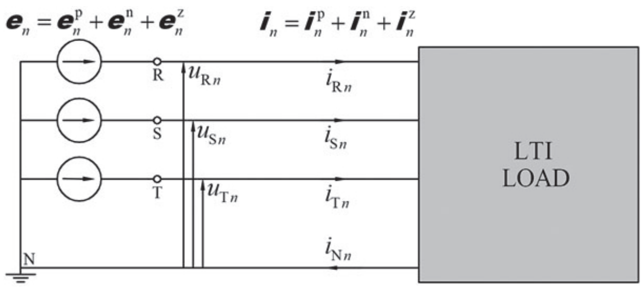

Fig. 1. LTI load supplied by a four-wire line

As we can see in Fig. 1, our voltage source could have symmetrical components, i.e. $\boldsymbol{e}_n = \boldsymbol{e}_n^{\mathrm{p}} + \boldsymbol{e}_n^{\mathrm{n}} + \boldsymbol{e}_n^{\mathrm{z}}$. If we assume that the vector of voltages has the same values as the voltages sources, it can be written that this vector is equal to $\boldsymbol{u}_n = \boldsymbol{u}_n^{\mathrm{p}} + \boldsymbol{u}_n^{\mathrm{n}} + \boldsymbol{u}_n^{\mathrm{z}}$. Currents can be described using symmetrical components, too.

The vector of the nonsinusoidal voltages, supplying an LTI unbalanced load, can be represented as follows:

$$\boldsymbol{u}(t) = \begin{bmatrix} u_{\mathrm{R}}(t) \\ u_{\mathrm{S}}(t) \\ u_{\mathrm{T}}(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re} \sum_{n \in N} \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re} \sum_{n \in N} \boldsymbol{U}_n e^{jn\omega_1 t}. \tag{1}$$

The line currents can be represented identically, namely:

$$\boldsymbol{i}(t)=\begin{bmatrix} i_{\mathrm{R}}(t) \\ i_{\mathrm{S}}(t) \\ i_{\mathrm{T}}(t) \end{bmatrix}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}\begin{bmatrix} \boldsymbol{I}_{\mathrm{R}n} \\ \boldsymbol{I}_{\mathrm{S}n} \\ \boldsymbol{I}_{\mathrm{T}n} \end{bmatrix}e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}\boldsymbol{I}_n e^{jn\omega_1 t}. \tag{2}$$

As mentioned above, the vector of voltages can have symmetrical components which are the sum of the positive, negative, and zero sequences components:

$$\begin{aligned}\boldsymbol{u}&=\sum_{n\in N}\left(\boldsymbol{u}_n^{\mathrm{p}}+\boldsymbol{u}_n^{\mathrm{n}}+\boldsymbol{u}_n^{\mathrm{z}}\right)\\ &=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}\left(\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{U}_n^{\mathrm{z}}\right)e^{jn\omega_1 t}\\ &=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}\left(\boldsymbol{1}^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{1}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{z}}\right)e^{jn\omega_1 t},\end{aligned} \tag{3}$$

where $\boldsymbol{U}_n^{\mathrm{p}}$, $\boldsymbol{U}_n^{\mathrm{n}}$ and $\boldsymbol{U}_n^{\mathrm{z}}$ are the complex RMS (crms) values of the symmetrical components of the positive, negative and zero sequences, described by the Fortescue Transformation [1, 2, 18, 22, 24]:

$$\begin{bmatrix} \boldsymbol{U}_n^{\mathrm{p}} \\ \boldsymbol{U}_n^{\mathrm{n}} \\ \boldsymbol{U}_n^{\mathrm{z}} \end{bmatrix}=\frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix}, \tag{4}$$

and where $\alpha=1e^{j120^\circ}$, $\alpha^*=1e^{-j120^\circ}$, and symbols:

$$\boldsymbol{1}^{\mathrm{p}}=\begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix}=\begin{bmatrix} 1 \\ e^{-j120^\circ} \\ e^{j120^\circ} \end{bmatrix}, \quad \boldsymbol{1}^{\mathrm{n}}=\begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix}=\begin{bmatrix} 1 \\ e^{j120^\circ} \\ e^{-j120^\circ} \end{bmatrix}, \quad \boldsymbol{1}^{\mathrm{z}}=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \tag{5}$$

denote unit symmetrical vectors of the positive – $\boldsymbol{1}^{\mathrm{p}}$, negative – $\boldsymbol{1}^{\mathrm{n}}$, and zero sequences – $\boldsymbol{1}^{\mathrm{z}}$, described in [4, 5].

Identically as in [14, 18, 25], the issue of the definition of apparent power S_n and complex apparent power $\boldsymbol{S}_n$ can be extended to the harmonics of higher orders and hence we can take a different symbol in this paper, which can be written as follows:

$$\boldsymbol{Y}_{\mathrm{b}n}=G_{\mathrm{b}n}+jB_{\mathrm{b}n}=\frac{P_n-jQ_n}{\|\boldsymbol{u}_n\|^2}=\frac{\boldsymbol{C}_n^*}{\|\boldsymbol{u}_n\|^2}, \tag{6}$$

where $\|\boldsymbol{u}_n\|$ means the three-phase RMS value of the voltages for each harmonic, which can be defined by the formulae:

$$\|\boldsymbol{u}_n\|=\sqrt{\frac{1}{T}\int_0^T \boldsymbol{u}_n^{\mathrm{T}}(t)\boldsymbol{u}_n(t)\,\mathrm{d}t}=\sqrt{U_{\mathrm{R}n}^2+U_{\mathrm{S}n}^2+U_{\mathrm{T}n}^2}, \tag{7}$$

where P_n and Q_n mean the three-phase values of the active and reactive powers for each harmonic.

If we add to each other every active power of each harmonic, then we can calculate the equivalent conductance of the whole system, as below:

$$G_{\mathrm{e}}=\frac{\sum\limits_{n\in N}P_n}{\sqrt{\sum\limits_{n\in N}\left(U_{\mathrm{R}n}^2+U_{\mathrm{S}n}^2+U_{\mathrm{T}n}^2\right)}}=\frac{P}{\|\boldsymbol{u}\|^2}, \tag{8}$$

which is a condition for the existence of the active current $\boldsymbol{i}_{\mathrm{a}}$ in the system, the waveform of which is:

$$\begin{aligned}\boldsymbol{i}_{\mathrm{a}}&=G_{\mathrm{e}}\boldsymbol{u}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}G_{\mathrm{e}}\boldsymbol{U}_n e^{jn\omega_1 t}\\ &=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}G_{\mathrm{e}}\left(\boldsymbol{1}^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{1}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{z}}\right)e^{jn\omega_1 t},\end{aligned} \tag{9}$$

and the three-phase RMS value of the active current is:

$$\|\boldsymbol{i}_{\mathrm{a}}\|=G_{\mathrm{e}}\|\boldsymbol{u}\|=\frac{P}{\|\boldsymbol{u}\|}. \tag{10}$$

We know from (6) that the active powers P_n of individual harmonics and the reactive powers Q_n of individual harmonics are associated with the equivalent conductances $G_{\mathrm{b}n}$ and the equivalent susceptances $B_{\mathrm{b}n}$ for those harmonics. Based on (6), (8), and (9), the description of the waveform of the scattered current $\boldsymbol{i}_{\mathrm{s}}$ takes the form:

$$\begin{aligned}\boldsymbol{i}_{\mathrm{s}}&=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}\left(G_{\mathrm{b}n}-G_{\mathrm{e}}\right)\boldsymbol{U}_n e^{jn\omega_1 t}\\ &=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}\left(G_{\mathrm{b}n}-G_{\mathrm{e}}\right)\left(\boldsymbol{1}^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{1}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{z}}\right)e^{jn\omega_1 t},\end{aligned} \tag{11}$$

where the three-phase RMS value is:

$$\|\boldsymbol{i}_{\mathrm{s}}\|=\sqrt{\sum_{n\in N}\left[\left(G_{\mathrm{b}n}-G_{\mathrm{e}}\right)^2\|\boldsymbol{u}_n\|^2\right]}. \tag{12}$$

By using the imaginary part from (6), the waveform of the reactive current $\boldsymbol{i}_{\mathrm{r}}$ can be represented as:

$$\begin{aligned}\boldsymbol{i}_{\mathrm{r}}&=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}jB_{\mathrm{b}n}\boldsymbol{U}_n e^{jn\omega_1 t}\\ &=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}jB_{\mathrm{b}n}\left(\boldsymbol{1}^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{1}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{z}}\right)e^{jn\omega_1 t},\end{aligned} \tag{13}$$

and its three-phase RMS value is:

$$\|\boldsymbol{i}_{\mathrm{r}}\|=\sqrt{\sum_{n\in N}\left[\left(B_{\mathrm{b}n}\right)^2\|\boldsymbol{u}_n\|^2\right]}. \tag{14}$$

Those currents are proportional to the supply voltage, and the supply voltage shifted in time by a quarter. Therefore, they have the same value of asymmetry as the supply voltage. However, the current of the original load $\boldsymbol{i}$, as a consequence of its unbalance, does not have the same asymmetry as the supply voltage $\boldsymbol{u}$. The current of the unbalanced load, powered from nonsinusoidal asymmetrical voltage source may have an unbalanced component:

$$\boldsymbol{i} - (\boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{r}}) = \boldsymbol{i} - \boldsymbol{i}_{\mathrm{b}} = \boldsymbol{i}_{\mathrm{u}}\,, \tag{15}$$

where its waveform takes the form:

$$\begin{aligned} \boldsymbol{i}_{\mathrm{u}} &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_{\mathrm{u}n} e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} (\boldsymbol{I}_n - \boldsymbol{I}_{\mathrm{b}n})\, e^{jn\omega_1 t} \\ &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} (\boldsymbol{I}_n - (G_{\mathrm{e}} + (G_{\mathrm{b}n} - G_{\mathrm{e}}) + jB_{\mathrm{b}n})\,\boldsymbol{U}_n)\, e^{jn\omega_1 t} \\ &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} ((\boldsymbol{I}_n - \boldsymbol{I}_{\mathrm{b}n})\,(\boldsymbol{1}^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{1}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}} + \boldsymbol{1}^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{z}}))\, e^{jn\omega_1 t}, \end{aligned} \tag{16}$$

and after transformation (15) we obtain:

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}\,, \tag{17}$$

which means that the nonsinusoidal current of the load is the sum of the active current, scattered current, reactive current and unbalanced current.

Each component is associated with a different physical phenomenon. The active current $\boldsymbol{i}_{\mathrm{a}}$ is related to the permanent flow of energy from the source to the load. The scattered current $\boldsymbol{i}_{\mathrm{s}}$ is linked with a change in conductance along with the order of harmonics. The reactive current $\boldsymbol{i}_{\mathrm{r}}$ is related to the displacement of the load current concerning the supply voltage. The unbalanced current $\boldsymbol{i}_{\mathrm{u}}$ is the result of the asymmetry of the currents caused by the unbalanced load and the voltages asymmetry. Due to the unambiguous connection of these currents with separate physical phenomena, they can be treated as the Currents' Physical Components (CPC Theory) of the load current.

The three-phase RMS value of the current of the load supplied from the nonsinusoidal asymmetrical voltage source is:

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_{\mathrm{a}}\|^2 + \|\boldsymbol{i}_{\mathrm{s}}\|^2 + \|\boldsymbol{i}_{\mathrm{r}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}\|^2\,. \tag{18}$$

Equation (18) is true on condition that the current components of the load are mutually orthogonal – which is presented in Appendix A.

3. Decomposition of the unbalanced current

The next step is the decomposition of the unbalanced current $\boldsymbol{i}_{\mathrm{u}}$ into three symmetrical components of the appropriate sequences.

We can present the equivalent admittance of the balanced load (6) by the parameters of the load and the supply voltages as shown below:

$$\begin{aligned} \boldsymbol{Y}_{\mathrm{b}n} &= G_{\mathrm{b}n} + jB_{\mathrm{b}n} = \frac{P - jQ}{\|\boldsymbol{u}_n\|^2} \\ &= \frac{\boldsymbol{Y}_{\mathrm{R}n}U_{\mathrm{R}n}^2 + \boldsymbol{Y}_{\mathrm{S}n}U_{\mathrm{S}n}^2 + \boldsymbol{Y}_{\mathrm{T}n}U_{\mathrm{T}n}^2}{\|\boldsymbol{u}_n\|^2}\,. \end{aligned} \tag{19}$$

Under the condition of the symmetry of the supply voltages, i.e. $U_{\mathrm{R}n} = U_{\mathrm{S}n} = U_{\mathrm{T}n}$, the equivalent admittance of the balanced load is:

$$\boldsymbol{Y}_{\mathrm{b}n} = \frac{1}{3}\,(\boldsymbol{Y}_{\mathrm{R}n} + \boldsymbol{Y}_{\mathrm{S}n} + \boldsymbol{Y}_{\mathrm{T}n}) = \boldsymbol{Y}_{\mathrm{e}n}\,, \tag{20}$$

and is called **the equivalent admittance of the load supplied from the nonsinusoidal symmetrical voltage source**.

The difference between admittances expressed in (19) and (20) is:

$$\begin{aligned} \boldsymbol{Y}_{\mathrm{d}n} &= G_{\mathrm{d}n} + jB_{\mathrm{d}n} = \boldsymbol{Y}_{\mathrm{e}n} - \boldsymbol{Y}_{\mathrm{b}n} \\ &= \frac{1}{3}\,(\boldsymbol{Y}_{\mathrm{R}n} + \boldsymbol{Y}_{\mathrm{S}n} + \boldsymbol{Y}_{\mathrm{T}n}) - \frac{\boldsymbol{Y}_{\mathrm{R}n}U_{\mathrm{R}n}^2 + \boldsymbol{Y}_{\mathrm{S}n}U_{\mathrm{S}n}^2 + \boldsymbol{Y}_{\mathrm{T}n}U_{\mathrm{T}n}^2}{\|\boldsymbol{u}_n\|^2}\,, \end{aligned} \tag{21}$$

and is called **the voltage asymmetry dependent admittance**.

In accordance with [26] in the four-wire systems supplied by waveforms of nonsinusoidal voltages, the unbalance of the load can be described by three admittances, i.e. **the unbalanced admittance of the positive sequence**, represented as:

$$\begin{aligned} \boldsymbol{A}_n^{\mathrm{p}} = \frac{1}{3}\big[&(\boldsymbol{Y}_{\mathrm{R}n} + \alpha\beta\boldsymbol{Y}_{\mathrm{S}n} + \alpha^*\beta^*\boldsymbol{Y}_{\mathrm{T}n}) \\ &- \boldsymbol{Y}_{\mathrm{e}n}\,(1 + \alpha\beta + \alpha^*\beta^*)\big], \end{aligned} \tag{22}$$

the unbalanced admittance of the negative sequence:

$$\begin{aligned} \boldsymbol{A}_n^{\mathrm{n}} = \frac{1}{3}\big[&(\boldsymbol{Y}_{\mathrm{R}n} + \alpha^*\beta\boldsymbol{Y}_{\mathrm{S}n} + \alpha\beta^*\boldsymbol{Y}_{\mathrm{T}n}) \\ &- \boldsymbol{Y}_{\mathrm{e}n}\,(1 + \alpha^*\beta + \alpha\beta^*)\big], \end{aligned} \tag{23}$$

and **the unbalanced admittance of the zero sequence:**

$$\boldsymbol{A}_n^{\mathrm{z}} = \frac{1}{3}\big[(\boldsymbol{Y}_{\mathrm{R}n} + \beta\boldsymbol{Y}_{\mathrm{S}n} + \beta^*\boldsymbol{Y}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{e}n}\,(1 + \beta + \beta^*)\big], \tag{24}$$

where the generalized rotation coefficient β has been described in [26] and is equal to:

$$\beta = (\alpha^*)^n = \begin{cases} 1 & \text{for } n = 3k \\ \alpha^* & \text{for } n = 3k+1 \\ \alpha & \text{for } n = 3k-1 \end{cases}. \tag{25}$$

Based on (19) and (21)–(24), the crms value of the R-line nonsinusoidal current can be represented as:

$$\begin{aligned} \boldsymbol{I}_{\mathrm{R}n} = {}&(\boldsymbol{Y}_{\mathrm{b}n} + \boldsymbol{Y}_{\mathrm{d}n})\boldsymbol{U}_{\mathrm{R}n} + (\boldsymbol{A}_n^{\mathrm{p}} + \boldsymbol{A}_n^{\mathrm{n}} + \boldsymbol{A}_n^{\mathrm{z}})\boldsymbol{U}_{\mathrm{R}n}^{\mathrm{p}} \\ &+ (\boldsymbol{A}_n^{\mathrm{p}} + \boldsymbol{A}_n^{\mathrm{n}} + \boldsymbol{A}_n^{\mathrm{z}})\boldsymbol{U}_{\mathrm{R}n}^{\mathrm{n}} + (\boldsymbol{A}_n^{\mathrm{p}} + \boldsymbol{A}_n^{\mathrm{n}} + \boldsymbol{A}_n^{\mathrm{z}})\boldsymbol{U}_{\mathrm{R}n}^{\mathrm{z}}\,. \end{aligned} \tag{26}$$

The crms current values of the nonsinusoidal current in the S-line can be expressed in the same way:

$$\begin{aligned} \boldsymbol{I}_{\mathrm{S}n} = {}&(\boldsymbol{Y}_{\mathrm{b}n} + \boldsymbol{Y}_{\mathrm{d}n})\boldsymbol{U}_{\mathrm{S}n} \\ &+ (\alpha^*\beta^*\boldsymbol{A}_n^{\mathrm{p}} + \alpha\beta^*\boldsymbol{A}_n^{\mathrm{n}} + \beta^*\boldsymbol{A}_n^{\mathrm{z}})\boldsymbol{U}_{\mathrm{S}n}^{\mathrm{p}} \\ &+ (\alpha^*\beta^*\boldsymbol{A}_n^{\mathrm{p}} + \alpha\beta^*\boldsymbol{A}_n^{\mathrm{n}} + \beta^*\boldsymbol{A}_n^{\mathrm{z}})\boldsymbol{U}_{\mathrm{S}n}^{\mathrm{n}} \\ &+ (\alpha^*\beta^*\boldsymbol{A}_n^{\mathrm{p}} + \alpha\beta^*\boldsymbol{A}_n^{\mathrm{n}} + \beta^*\boldsymbol{A}_n^{\mathrm{z}})\boldsymbol{U}_{\mathrm{S}n}^{\mathrm{z}}\,, \end{aligned} \tag{27}$$

and the T-line nonsinusoidal current:

$$\begin{aligned}\boldsymbol{I}_{\mathrm{T}n} &= \left(\boldsymbol{Y}_{\mathrm{b}n}+\boldsymbol{Y}_{\mathrm{d}n}\right)\boldsymbol{U}_{\mathrm{T}n}\\ &+\left(\alpha\beta\boldsymbol{A}_n^{\mathrm{p}}+\alpha^*\beta\boldsymbol{A}_n^{\mathrm{n}}+\beta\boldsymbol{A}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{T}n}^{\mathrm{p}}\\ &+\left(\alpha\beta\boldsymbol{A}_n^{\mathrm{p}}+\alpha^*\beta\boldsymbol{A}_n^{\mathrm{n}}+\beta\boldsymbol{A}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{T}n}^{\mathrm{n}}\\ &+\left(\alpha\beta\boldsymbol{A}_n^{\mathrm{p}}+\alpha^*\beta\boldsymbol{A}_n^{\mathrm{n}}+\beta\boldsymbol{A}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{T}n}^{\mathrm{z}},\end{aligned} \tag{28}$$

respectively.

It is possible to simplify the expression of all three crms values of the line currents. For this purpose, expressions (26)–(28) can be transformed using the following simplifications, in which it can be noted that:

1. For harmonics of the positive sequences, the unbalanced admittance of the positive sequence is always equal to 0, for harmonics of the negative sequences the unbalanced admittance of the negative sequence is always equal to 0, and for harmonics of the zero sequences, the unbalanced admittance of the zero sequence is always equal to 0, therefore:

$$\left.\begin{cases}\boldsymbol{A}_n^{\mathrm{p}}=0 & \text{for } n=3k{+}1\\ \boldsymbol{A}_n^{\mathrm{n}}=0 & \text{for } n=3k{-}1\\ \boldsymbol{A}_n^{\mathrm{z}}=0 & \text{for } n=3k\end{cases}\right\}=\boldsymbol{Y}_n^{\mathrm{p}}, \tag{29}$$

and it is called **the generalized unbalanced admittance of the positive sequence**.

2. For harmonics of the positive sequences, the unbalanced admittance of the negative sequence is always different from 0, for harmonics of the negative sequences, the unbalanced admittance of the zero sequence is always different from 0, and for harmonics of the zero sequences, the unbalanced admittance of the positive sequence is always different from 0, therefore:

$$\left.\begin{cases}\boldsymbol{A}_n^{\mathrm{n}}\neq 0 & \text{for } n=3k{+}1\\ \boldsymbol{A}_n^{\mathrm{z}}\neq 0 & \text{for } n=3k{-}1\\ \boldsymbol{A}_n^{\mathrm{p}}\neq 0 & \text{for } n=3k\end{cases}\right\}=\boldsymbol{Y}_n^{\mathrm{n}}, \tag{30}$$

and it is called **the generalized unbalanced admittance of the negative sequence**.

3. For harmonics of the positive sequences, the unbalanced admittance of the zero sequence is always different from 0, for harmonics of the negative sequences, the unbalanced admittance of the positive sequence is always different from 0, and for harmonics of the zero sequences, the unbalanced admittance of the negative sequence is always different from 0, therefore:

$$\left.\begin{cases}\boldsymbol{A}_n^{\mathrm{z}}\neq 0 & \text{for } n=3k{+}1\\ \boldsymbol{A}_n^{\mathrm{p}}\neq 0 & \text{for } n=3k{-}1\\ \boldsymbol{A}_n^{\mathrm{n}}\neq 0 & \text{for } n=3k\end{cases}\right\}=\boldsymbol{Y}_n^{\mathrm{z}}. \tag{31}$$

After considering the generalized unbalanced admittances described by (29)–(31), equations (26)–(28) can be presented as follows:

1. The crms value of the R-line nonsinusoidal current:

$$\begin{aligned}\boldsymbol{I}_{\mathrm{R}n} &= \left(\boldsymbol{Y}_{\mathrm{b}n}+\boldsymbol{Y}_{\mathrm{d}n}\right)\boldsymbol{U}_{\mathrm{R}n}+\left(\boldsymbol{Y}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{R}n}^{\mathrm{p}}\\ &+\left(\boldsymbol{Y}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{R}n}^{\mathrm{n}}+\left(\boldsymbol{Y}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{R}n}^{\mathrm{z}}.\end{aligned} \tag{32}$$

2. The crms value of the S-line nonsinusoidal current:

$$\begin{aligned}\boldsymbol{I}_{\mathrm{S}n} &= \left(\boldsymbol{Y}_{\mathrm{b}n}+\boldsymbol{Y}_{\mathrm{d}n}\right)\boldsymbol{U}_{\mathrm{S}n}+\left(\boldsymbol{Y}_n^{\mathrm{p}}+\alpha^*\boldsymbol{Y}_n^{\mathrm{n}}+\alpha\boldsymbol{Y}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{S}n}^{\mathrm{p}}\\ &+\left(\boldsymbol{Y}_n^{\mathrm{p}}+\alpha^*\boldsymbol{Y}_n^{\mathrm{n}}+\alpha\boldsymbol{Y}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{S}n}^{\mathrm{n}}\\ &+\left(\boldsymbol{Y}_n^{\mathrm{p}}+\alpha^*\boldsymbol{Y}_n^{\mathrm{n}}+\alpha\boldsymbol{Y}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{S}n}^{\mathrm{z}}.\end{aligned} \tag{33}$$

3. The crms value of the T-line nonsinusoidal current:

$$\begin{aligned}\boldsymbol{I}_{\mathrm{T}n} &= \left(\boldsymbol{Y}_{\mathrm{b}n}+\boldsymbol{Y}_{\mathrm{d}n}\right)\boldsymbol{U}_{\mathrm{T}n}+\left(\boldsymbol{Y}_n^{\mathrm{p}}+\alpha\boldsymbol{Y}_n^{\mathrm{n}}+\alpha^*\boldsymbol{Y}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{T}n}^{\mathrm{p}}\\ &+\left(\boldsymbol{Y}_n^{\mathrm{p}}+\alpha\boldsymbol{Y}_n^{\mathrm{n}}+\alpha^*\boldsymbol{Y}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{T}n}^{\mathrm{n}}\\ &+\left(\boldsymbol{Y}_n^{\mathrm{p}}+\alpha\boldsymbol{Y}_n^{\mathrm{n}}+\alpha^*\boldsymbol{Y}_n^{\mathrm{z}}\right)\boldsymbol{U}_{\mathrm{T}n}^{\mathrm{z}}.\end{aligned} \tag{34}$$

Combining (32)–(34) into one formula, we receive a vector of the three-phase crms values of the nonsinusoidal line currents of the load:

$$\begin{aligned}\boldsymbol{I} &= \begin{bmatrix}\boldsymbol{I}_{\mathrm{R}n}\\ \boldsymbol{I}_{\mathrm{S}n}\\ \boldsymbol{I}_{\mathrm{T}n}\end{bmatrix} = G_{\mathrm{e}}\boldsymbol{U}_n+jB_{\mathrm{b}n}\boldsymbol{U}_n+\left(G_{\mathrm{b}n}-G_{\mathrm{e}}\right)\boldsymbol{U}_n\\ &+\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n+\boldsymbol{1}^{\mathrm{p}}\left(\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{z}}\right)\\ &+\boldsymbol{1}^{\mathrm{n}}\left(\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{z}}\right)\\ &+\boldsymbol{1}^{\mathrm{z}}\left(\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{z}}\right),\end{aligned} \tag{35}$$

where the three-phase value of the unbalanced $\boldsymbol{I}_{\mathrm{u}}$ current is:

$$\begin{aligned}\boldsymbol{I}_{\mathrm{u}} &= \boldsymbol{Y}_{\mathrm{d}n}U_n+\boldsymbol{1}^{\mathrm{p}}\left(\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{z}}\right)\\ &+\boldsymbol{1}^{\mathrm{n}}\left(\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{z}}\right)\\ &+\boldsymbol{1}^{\mathrm{z}}\left(\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{z}}\right)\\ &= \boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n+\boldsymbol{J}_n^{\mathrm{p}}+\boldsymbol{J}_n^{\mathrm{n}}+\boldsymbol{J}_n^{\mathrm{z}},\end{aligned} \tag{36}$$

where the sources of currents $\boldsymbol{J}_n^{\mathrm{p}}$, $\boldsymbol{J}_n^{\mathrm{n}}$, $\boldsymbol{J}_n^{\mathrm{z}}$ represent the currents of the positive, negative and zero sequences for each harmonic.

Their vectors of the crms values are described below:

$$\begin{aligned}\boldsymbol{J}_n^{\mathrm{p}} &= \boldsymbol{1}^{\mathrm{p}}\left(\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{z}}\right),\\ \boldsymbol{J}_n^{\mathrm{n}} &= \boldsymbol{1}^{\mathrm{n}}\left(\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{z}}\right),\\ \boldsymbol{J}_n^{\mathrm{z}} &= \boldsymbol{1}^{\mathrm{z}}\left(\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{z}}\right).\end{aligned} \tag{37}$$

The relationship (37) can be subjected to further transformations, then the vector of crms values takes the form:

$$\begin{aligned}\boldsymbol{I}_{\mathrm{u}} &= \boldsymbol{1}^{\mathrm{p}}\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{1}^{\mathrm{n}}\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{z}}\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^{\mathrm{z}}\\ &+\boldsymbol{1}^{\mathrm{p}}\left(\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{z}}\right)\\ &+\boldsymbol{1}^{\mathrm{n}}\left(\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{z}}\right)\\ &+\boldsymbol{1}^{\mathrm{z}}\left(\boldsymbol{Y}_n^{\mathrm{z}}\boldsymbol{U}_n^{\mathrm{p}}+\boldsymbol{Y}_n^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}}+\boldsymbol{Y}_n^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{z}}\right)=\boldsymbol{I}_{\mathrm{u}n}^{\mathrm{p}}+\boldsymbol{I}_{\mathrm{u}n}^{\mathrm{n}}+\boldsymbol{I}_{\mathrm{u}n}^{\mathrm{z}}.\end{aligned} \tag{38}$$

From the relationship above, we can extract three unbalanced currents of the respective sequences, namely:

1. The waveform of the positive sequence unbalanced current $\boldsymbol{i}_\mathrm{u}^\mathrm{p}$:

$$\begin{aligned} \boldsymbol{i}_\mathrm{u}^\mathrm{p} &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_{\mathrm{u}n}^\mathrm{p} e^{jn\omega_1 t} \\ &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{1}^\mathrm{p}\big(\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^\mathrm{p} + \boldsymbol{Y}_n^\mathrm{p}\boldsymbol{U}_n^\mathrm{p} + \boldsymbol{Y}_n^\mathrm{z}\boldsymbol{U}_n^\mathrm{n} \\ &\quad + \boldsymbol{Y}_n^\mathrm{n}\boldsymbol{U}_n^\mathrm{z}\big) e^{jn\omega_1 t}. \end{aligned} \tag{39}$$

2. The waveform of the negative sequence unbalanced current $\boldsymbol{i}_\mathrm{u}^\mathrm{n}$:

$$\begin{aligned} \boldsymbol{i}_\mathrm{u}^\mathrm{n} &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_{\mathrm{u}n}^\mathrm{n} e^{jn\omega_1 t} \\ &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{1}^\mathrm{n}\big(\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^\mathrm{n} + \boldsymbol{Y}_n^\mathrm{n}\boldsymbol{U}_n^\mathrm{p} + \boldsymbol{Y}_n^\mathrm{p}\boldsymbol{U}_n^\mathrm{n} \\ &\quad + \boldsymbol{Y}_n^\mathrm{z}\boldsymbol{U}_n^\mathrm{z}\big) e^{jn\omega_1 t}. \end{aligned} \tag{40}$$

3. The waveform of the zero sequence unbalanced current $\boldsymbol{i}_\mathrm{u}^\mathrm{z}$:

$$\begin{aligned} \boldsymbol{i}_\mathrm{u}^\mathrm{z} &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_{\mathrm{u}n}^\mathrm{z} e^{jn\omega_1 t} \\ &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{1}^\mathrm{z}\big(\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^\mathrm{z} + \boldsymbol{Y}_n^\mathrm{z}\boldsymbol{U}_n^\mathrm{p} + \boldsymbol{Y}_n^\mathrm{n}\boldsymbol{U}_n^\mathrm{n} \\ &\quad + \boldsymbol{Y}_n^\mathrm{p}\boldsymbol{U}_n^\mathrm{z}\big) e^{jn\omega_1 t}. \end{aligned} \tag{41}$$

The three-phase RMS values of the unbalanced currents of respective sequences are presented below:

1. The three-phase RMS value of the positive sequence unbalanced current:

$$\|\boldsymbol{i}_\mathrm{u}^\mathrm{p}\| = \sqrt{3\cdot\sum_{n\in N}\left|\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^\mathrm{p} + \boldsymbol{Y}_n^\mathrm{p}\boldsymbol{U}_n^\mathrm{p} + \boldsymbol{Y}_n^\mathrm{z}\boldsymbol{U}_n^\mathrm{n} + \boldsymbol{Y}_n^\mathrm{n}\boldsymbol{U}_n^\mathrm{z}\right|^2}. \tag{42}$$

2. The three-phase RMS value of the negative sequence unbalanced current:

$$\|\boldsymbol{i}_\mathrm{u}^\mathrm{n}\| = \sqrt{3\cdot\sum_{n\in N}\left|\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^\mathrm{n} + \boldsymbol{Y}_n^\mathrm{n}\boldsymbol{U}_n^\mathrm{p} + \boldsymbol{Y}_n^\mathrm{p}\boldsymbol{U}_n^\mathrm{n} + \boldsymbol{Y}_n^\mathrm{z}\boldsymbol{U}_n^\mathrm{z}\right|^2}. \tag{43}$$

3. The three-phase RMS value of the zero sequence unbalanced current:

$$\|\boldsymbol{i}_\mathrm{u}^\mathrm{z}\| = \sqrt{3\cdot\sum_{n\in N}\left|\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^\mathrm{z} + \boldsymbol{Y}_n^\mathrm{z}\boldsymbol{U}_n^\mathrm{p} + \boldsymbol{Y}_n^\mathrm{n}\boldsymbol{U}_n^\mathrm{n} + \boldsymbol{Y}_n^\mathrm{p}\boldsymbol{U}_n^\mathrm{z}\right|^2}. \tag{44}$$

In summary, the load's current can be built of six orthogonal components:

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{s}\|^2 + \|\boldsymbol{i}_\mathrm{r}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{p}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{n}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{z}\|^2. \tag{45}$$

All of the components of the current described above are mutually orthogonal – Appendix A.

By multiplying the individual components in (45) by the square of the three-phase RMS value of the voltage supply $\|\boldsymbol{u}\|^2$, the power equation of the load powered from a nonsinusoidal asymmetrical voltage source takes the form:

$$S^2 = P^2 + Q^2 + D_\mathrm{s}^2 + D_\mathrm{u}^{\mathrm{p}2} + D_\mathrm{u}^{\mathrm{n}2} + D_\mathrm{u}^{\mathrm{z}2}. \tag{46}$$

In (46) respective powers are described as follows:

1. The apparent power S:

$$S = \|\boldsymbol{u}\|\,\|\boldsymbol{i}\|. \tag{47}$$

2. The active power P:

$$P = \|\boldsymbol{u}\|\,\|\boldsymbol{i}_\mathrm{a}\|. \tag{48}$$

3. The reactive power Q:

$$Q = \|\boldsymbol{u}\|\,\|\boldsymbol{i}_\mathrm{r}\|. \tag{49}$$

4. The scattered power D_s:

$$D_\mathrm{s} = \|\boldsymbol{u}\|\,\|\boldsymbol{i}_\mathrm{s}\|. \tag{50}$$

5. The positive sequence unbalanced power D_u^p:

$$D_\mathrm{u}^\mathrm{p} = \|\boldsymbol{u}\|\,\|\boldsymbol{i}_\mathrm{u}^\mathrm{p}\|. \tag{51}$$

6. The negative sequence unbalanced power D_u^n:

$$D_\mathrm{u}^\mathrm{n} = \|\boldsymbol{u}\|\,\|\boldsymbol{i}_\mathrm{u}^\mathrm{n}\|. \tag{52}$$

7. The zero sequence unbalanced power D_u^z:

$$D_\mathrm{u}^\mathrm{z} = \|\boldsymbol{u}\|\,\|\boldsymbol{i}_\mathrm{u}^\mathrm{z}\|. \tag{53}$$

Through the components (48)–(53) and the apparent power S (47) of the unbalanced load supplied with asymmetrical nonsinusoidal voltages, the power factor λ can be expressed as:

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2 + D_\mathrm{s}^2 + D_\mathrm{u}^{\mathrm{p}2} + D_\mathrm{u}^{\mathrm{n}2} + D_\mathrm{u}^{\mathrm{z}2}}}. \tag{54}$$

As we can see from (54), the power factor of the system under consideration depends on six powers that together form the apparent power S.

4. Theoretical illustration

For theoretical illustration, the three-phase four-wire system supplied from the asymmetrical nonsinusoidal voltage source has been chosen. The circuit is presented in Fig. 2. All calculations are based on the assumption of LTI load. Besides, the supply voltage is unchanged in time and it is a source that generates the fundamental harmonic, its frequency is 50 Hz, and the 3rd and 5th harmonic.

The values of the phase impedances of the load for the fundamental harmonic, shown in Fig. 2, are compiled in Table 1.

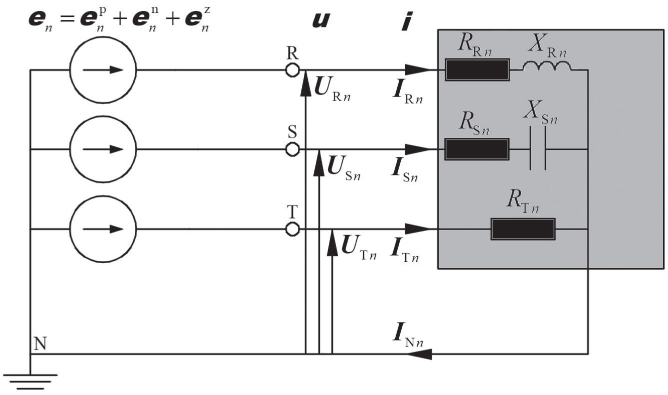

Fig. 2. Three-phase four-wire system with LTI load

Table 1
List of phase impedances in ohms with division into resistance, inductive and capacitive reactance values

Line	R	X_L	X_C
R	1	1	–
S	2	–	2
T	0.5	–	–

Table 2 presents the phase values of the nonsinusoidal voltages supply as well as the values of the nonsinusoidal line currents of the load.

The three-phase waveforms of the voltages, at the load terminals, are shown in Fig. 3.

Table 2
List of the values of the phase voltages and the values of the line currents

Harmonic order	Phase R	Phase S	Phase T
	U_n in [V]		
1st	$230e^{j7°}$	$225e^{-j110°}$	$200e^{j103°}$
3rd	$20e^{-j5°}$	$23e^{j10°}$	$90e^{-j10°}$
5th	$60e^{j6°}$	$22e^{j114°}$	$67e^{-j80°}$
	I_n in [A]		
1st	$162.64e^{-j38°}$	$79.55e^{-j65°}$	$400e^{j103°}$
3rd	$6.33e^{-j76.6°}$	$10.91e^{j28.4°}$	$180e^{-j10°}$
5th	$11.77e^{-j72.7°}$	$10.79e^{j125.3°}$	$134e^{-j80°}$

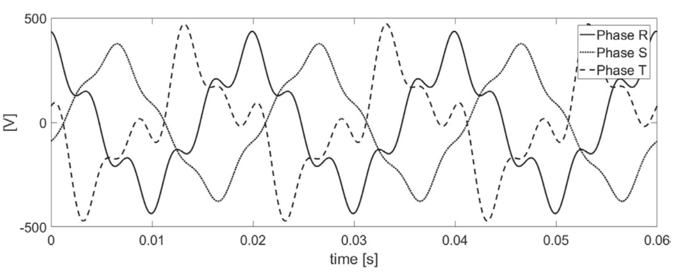

Fig. 3. The three-phase voltages waveforms at the terminals of the load

The load currents waveforms are shown in Fig. 4.

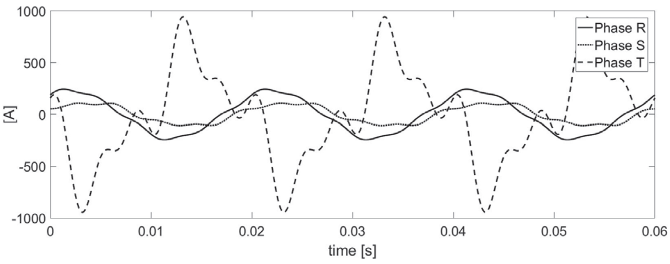

Fig. 4. The load currents waveforms

Table 3 presents the values of individual admittances for appropriate harmonics described in CPC Theory (6), (8), (21)–(24).

Table 3
List of the values of the individual admittances

Parameter [S]	Harmonic order		
	1st	3rd	5th
G_e	0.899		
Y_{dn}	$0.088e^{j8.4°}$	$0.976e^{-j177.3°}$	$0.255e^{j170.2°}$
Y_{bn}	$0.835e^{-j6.6°}$	$1.825e^{-j0.1°}$	$1.093e^{-j4°}$
A_n^p	–	$0.709e^{-j126.2°}$	$0.517e^{j136.1°}$
A_n^n	$0.767e^{-j111.5°}$	$0.463e^{j135.8°}$	–
A_n^z	$0.327e^{j114.6°}$	–	$0.673e^{-j129.6°}$

The three-phase RMS value $\|\boldsymbol{u}\|$ of the supplying voltages (7) and the three-phase RMS value $\|\boldsymbol{i}\|$ of the line currents are equal to the values for all harmonics:

$$\|\boldsymbol{u}\| = 401.41 \text{ V}, \quad \|\boldsymbol{i}\| = 493.51 \text{ A}.$$

Table 4 presents the three-phase RMS values of currents' components for appropriate harmonics described in CPC Theory (10), (12), (14), (42)–(44).

Table 4
List of the values of the currents' components

Current [A]	Harmonic order			Total value
	1st	3rd	5th	
$\|\boldsymbol{i}_a\|$	340.77	85.47	83.29	361.06
$\|\boldsymbol{i}_s\|$	26.38	87.94	17.69	93.50
$\|\boldsymbol{i}_r\|$	36.41	0.43	6.97	37.07
$\|\boldsymbol{i}_u^p\|$	57.87	28.72	63.89	90.86
$\|\boldsymbol{i}_u^n\|$	270.71	27.84	27.04	273.48
$\|\boldsymbol{i}_u^z\|$	126.40	29.78	56.16	141.48

By using (45) we obtain the three-phase RMS value of the currents of the load:

$$\begin{aligned}\|\boldsymbol{i}\| &= \\ &= \sqrt{361.06^2+93.5^2+37.07^2+90.86^2+273.48^2+141.48^2} \\ &= 493.51 \text{ A}.\end{aligned}$$

.

Based on (47)–(53), we can calculate the power components, which will be used to calculate the power factor (54). The power factor has the value:

$$\lambda = \frac{144933.5}{198096.3} = 0.732\,.$$

To sum up, the three-phase value of the components of currents of the load, in accordance with the CPC Theory, is the same as the three-phase RMS value calculated traditionally. Moreover, the power factor of the system depends on six powers and not only on the active and reactive powers.

5. Conclusion

This article shows that the physical components of the load current are associated with specific physical phenomena and can be described in the asymmetry of the voltage supply even if the voltage is nonsinusoidal.

The equations shown in the publication allow describing three-phase four-wire circuits with the asymmetrical nonsinusoidal voltage supply, which is another step in the development of the Currents' Physical Components Theory, until then described only with symmetrical power supply.

Furthermore, this is the commencement of the description of the possibility to determine the parameters of the balancing reactance compensator in asymmetrical four-wire systems supplied from a nonsinusoidal voltage source, because each of the currents' components is mutually orthogonal.

Appendix A

Vectors are mutually orthogonal when the three-phase scalar product [1, 2] is equal to zero:

$$(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{\mathrm{T}}\int_0^T \boldsymbol{x}(t)^{\mathrm{T}} \times \boldsymbol{y}(t)\,\mathrm{d}t = 0. \tag{A1}$$

If three-phase quantities are expressed in the form of complex RMS values, namely:

$$\boldsymbol{x} = \sqrt{2}\,\mathrm{Re}\left\{\boldsymbol{X}e^{j\omega t}\right\}, \quad \boldsymbol{y} = \sqrt{2}\,\mathrm{Re}\left\{\boldsymbol{Y}e^{j\omega t}\right\}, \tag{A2}$$

then their scalar product is equal to:

$$\begin{aligned}(\boldsymbol{x},\boldsymbol{y}) &= \frac{2}{\mathrm{T}}\int_0^T \mathrm{Re}\left\{\boldsymbol{X}^{\mathrm{T}}e^{j\omega t}\right\}\,\mathrm{Re}\left\{\boldsymbol{Y}e^{j\omega t}\right\}\mathrm{d}t \\ &= \mathrm{Re}\left\{\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Y}^*\right\}.\end{aligned} \tag{A3}$$

The above assumptions are true for any harmonic, so it is sufficient to prove orthogonality between the following components:

- $(\boldsymbol{i}_{\mathrm{a}},\ \boldsymbol{i}_{\mathrm{s}})$:

$$\begin{aligned}(\boldsymbol{i}_{\mathrm{a}},\ \boldsymbol{i}_{\mathrm{s}}) &= \mathrm{Re}\sum_{n\in N}\left(\left(G_{\mathrm{b}n}-G_{\mathrm{e}}\right)\boldsymbol{U}_n^{\mathrm{T}}\left(G_{\mathrm{e}}\boldsymbol{U}_n\right)^*\right) \\ &= \mathrm{Re}\sum_{n\in N}\left(\left(G_{\mathrm{b}n}\boldsymbol{U}_n^{\mathrm{T}}-G_{\mathrm{e}}\boldsymbol{U}_n^{\mathrm{T}}\right)G_{\mathrm{e}}\boldsymbol{U}_n^*\right) \\ &= \mathrm{Re}\sum_{n\in N}G_{\mathrm{e}}\left(G_{\mathrm{b}n}\boldsymbol{U}_n^{\mathrm{T}}\boldsymbol{U}_n^*-G_{\mathrm{e}}\boldsymbol{U}_n^{\mathrm{T}}\boldsymbol{U}_n^*\right) \\ &= \mathrm{Re}\sum_{n\in N}G_{\mathrm{e}}\left(G_{\mathrm{b}n}\|\boldsymbol{u}_n\|^2-G_{\mathrm{e}}\|\boldsymbol{u}_n\|^2\right) \\ &= G_{\mathrm{e}}\left(\sum_{n\in N}G_{\mathrm{b}n}\|\boldsymbol{u}_n\|^2-G_{\mathrm{e}}\sum_{n\in N}\|\boldsymbol{u}_n\|^2\right) \\ &= G_{\mathrm{e}}(P-P)=0.\end{aligned}$$

- $(\boldsymbol{i}_{\mathrm{s}},\ \boldsymbol{i}_{\mathrm{r}})$:

$$\begin{aligned}(\boldsymbol{i}_{\mathrm{s}},\ \boldsymbol{i}_{\mathrm{r}}) &= \mathrm{Re}\sum_{n\in N}\left(\left(G_{\mathrm{b}n}-G_{\mathrm{e}}\right)\boldsymbol{U}_n^{\mathrm{T}}\left(jB_{\mathrm{b}n}\boldsymbol{U}_n\right)^*\right) \\ &= \mathrm{Re}\sum_{n\in N}\left(\left(G_{\mathrm{b}n}\boldsymbol{U}_n^{\mathrm{T}}-G_{\mathrm{e}}\boldsymbol{U}_n^{\mathrm{T}}\right)-jB_{\mathrm{b}n}^*\boldsymbol{U}_n^*\right) \\ &= \mathrm{Re}\left\{-j\sum_{n\in N}\left(\left(G_{\mathrm{b}n}-G_{\mathrm{e}}\right)B_{\mathrm{b}n}\right)\|\boldsymbol{u}_n\|^2\right\}=0.\end{aligned}$$

- $(\boldsymbol{i}_{\mathrm{s}},\ \boldsymbol{i}_{\mathrm{u}})$:

$$\begin{aligned}(\boldsymbol{i}_{\mathrm{s}},\ \boldsymbol{i}_{\mathrm{u}}) &= \mathrm{Re}\sum_{n\in N}\left(\left(G_{\mathrm{b}n}-G_{\mathrm{e}}\right)\boldsymbol{U}_n\right)^{\mathrm{T}}\cdot\left(\boldsymbol{I}_n-\boldsymbol{I}_{\mathrm{b}n}\right)^* \\ &= \mathrm{Re}\left\{\sum_{n\in N}\left(G_{\mathrm{b}n}\boldsymbol{U}_n^{\mathrm{T}}-G_{\mathrm{e}}\boldsymbol{U}_n^{\mathrm{T}}\right)\left(\boldsymbol{I}_n^*-\boldsymbol{Y}_{\mathrm{b}n}^*\boldsymbol{U}_n^*\right)\right\} \\ &= \mathrm{Re}\left\{\sum_{n\in N}\left[G_{\mathrm{b}n}\boldsymbol{I}_n^*\boldsymbol{U}_n^{\mathrm{T}}-G_{\mathrm{e}}\boldsymbol{I}_n^*\boldsymbol{U}_n^{\mathrm{T}}-G_{\mathrm{b}n}\boldsymbol{Y}_{\mathrm{b}n}^*\boldsymbol{U}_n^{\mathrm{T}}\boldsymbol{U}_n^*\right.\right. \\ &\qquad \left.\left.+\,G_{\mathrm{e}}\boldsymbol{Y}_{\mathrm{b}n}^*\boldsymbol{U}_n^{\mathrm{T}}\boldsymbol{U}_n^*\right]\right\} \\ &= \mathrm{Re}\left\{\sum_{n\in N}G_{\mathrm{b}n}\boldsymbol{C}^*-\sum_{n\in N}G_{\mathrm{b}n}\boldsymbol{C}^*+G_{\mathrm{e}}\sum_{n\in N}\boldsymbol{C}^*-G_{\mathrm{e}}\sum_{n\in N}\boldsymbol{C}^*\right\} \\ &= 0.\end{aligned}$$

With respect to the fact that all currents are mutually orthogonal, relationship (54) is accomplished.

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